Approved Prep Provider CFA Institute

# 2017 Level I Formulas

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# Formula of Formulas

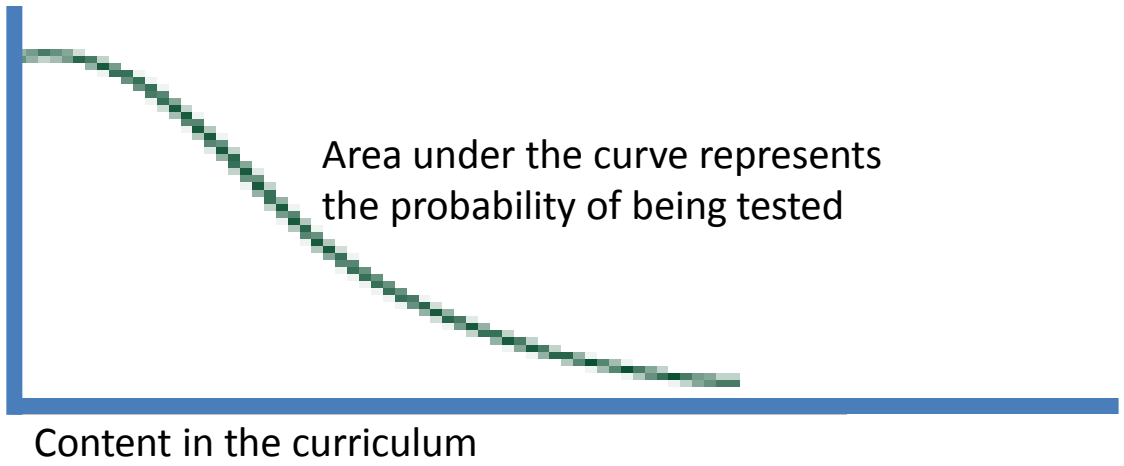
Have to know

**Type 1: Formula exists, but what really matters is the intuition**

**Type 2: Know the formula, good to know the intuition**

Should know

Type 3: Learn the formula, don't worry about the intuition




Nice to know

Type 4: Difficult formula and probability of being tested is low

# Quant: TVM

Interest rate = Real risk-free rate + Inflation premium + Default risk premium + Liquidity premium + Maturity premium

$FV_N = PV\ (1 + r)^N$

$FV_N = PV\ e^{r\,N}$

EAR = $(1 + \text{Periodic interest rate})^m - 1$

EAR = $e^r - 1$

Annuity formulas exist but use the calculator

PV of a perpetuity = A/r

# Quant: DCF Applications

$NPV = \sum [CF_t / (1+r)^t]$

IRR is the rate which makes NPV = 0

Bank Discount Yield = $(D/F) \times 360/t$

Holding Period Yield = $(P_1 - P_0 + D) / P_0$

Money Market Yield = HPY x 360 / t

Effective Annual Yield = $(1 + HPY)^{365/t} - 1$

Effective Annual Return = $(1 + \text{Periodic interest rate})^m - 1$

# Quant: Statistics

Geometric Mean = $[(1+R_1)(1+R_2).....(1+R_n)]^{1/n} - 1$

Harmonic Mean = $n / \sum (1/X_i)$

Weighted Mean = $\sum w_i X_i$

Location of observation at yth percentile: $L_y = (n + 1) (y/100)$

**Population and sample variance: use the calculator**

MAD = average of the absolute values of deviations from the mean

Range = maximum value – minimum value

Chebyshev's inequality states that for any set of observations, the proportion of the observations within k standard deviations of the mean is at least: $1 – (1/k^2)$ for all $k > 1$

Coefficient of variation = Risk / Return

Sharpe ratio = Excess return / Risk

Excess Kurtosis = Sample Kurtosis - 3

# Quant: Probability

Multiplication rule: $P(AB) = P(A|B) \times P(B)$

Addition rule: $P(A \text{ or } B) = P(A) + P(B) - P(AB)$

Total probability rule: $P(A) = P(AS) + P(AS^C) = P(A|S)\,P(S) + P(A|S^C)\,P(S^C)$

$P(E \mid I) = P(E) \times P(I|E) / P(I)$

$Cov(R_i, R_j) = E[(R_i - ER_i)(R_j - ER_j)]$

$\rho(R_i, R_j) = Cov(R_i, R_j) / \sigma(R_i)\,\sigma(R_j)$

$E(R_P) = w_1R_1 + w_2R_2$

$\sigma^2(R_P) = w_1^2\sigma_1^2(R_1) + w_2^2\sigma_2^2(R_2) + 2w_1w_2Cov(R_1R_2)$

${}_nP_r = n! / (n - r)!$

${}_nC_r = n! / (n - r)!r!$

# Quant: Distributions, Estimation, Hypothesis Testing

Binomial random variable: $p(x) = P(X = x) = {}_nC_x\ p^x (1 - p)^{n-x}$

Expected value = np and variance = n p (1 – p)

Normal distribution to standard normal: $z = (X - \mu) / \sigma$

SFRatio = $[E(R_p) - R_L] / \sigma_p$

Standard error of sample mean = $\sigma_X = \sigma / \sqrt{n}$ or $s_X = s / \sqrt{n}$

Confidence Interval = $X \pm z_{\alpha/2}(\sigma / \sqrt{n})$

Test statistic when testing for population mean: $\dfrac{\bar{X}_{RP} - \mu_0}{s_{\bar{x}}}$

# Economics

Demand function
Inverse demand function

$$Q_A = 2 - 0.4\,P_A + 0.0005\,I + 0.10\,P_B - 0.15\,P_C$$

Supply function and inverse demand function

Consumer surplus

Producer surplus

Total surplus

Elasticity = % change in quantity demanded / % change in price

Elasticity of Demand = $\%\Delta Q / \%\Delta P = (\Delta Q / \Delta P) \times P/Q$

- Own price
- Substitute
- Complement
- Income

Flatter curve: more elastic

Top left: more elastic

# Economics

Economic profit = Accounting profit – Total implicit opportunity costs

Economic profit = Total revenue – Total economic costs

Profit is maximized when MR = MC

Quantity, Price, Marginal Revenue

- $Q = 50 – 2P$
- $P = 25 – 0.5\ Q$
- $TR = PQ = 25\ Q – 0.5\ Q^2$
- $MR = 25 - Q_P$

In perfectly competitive markets:
P = MR = AR = D

In monopolistic markets:
MR = P [1 – 1/E]
Profit maximization condition: MR = MC
MC = P [1 – 1/E]
Profit maximizing price = MC / [1 – 1/E]

# Economics

Aggregate Expenditure = Aggregate Output = Aggregate Income

**GDP Deflator = (Nominal GDP / Real GDP) x 100**

**GDP based on expenditure approach =** Consumer spending on goods and services + Business gross fixed investment + Change in inventories + Government spending on goods and services + Government gross fixed investment + Exports − Imports + Statistical discrepancy

**GDP based on income approach =** National income + Capital consumption allowance + Statistical discrepancy

**National income =** Compensation of employees + Corporate profits before taxes + Interest income + Unincorporated business net income + Rent + Indirect business taxes less subsidies

**Personal income =** National income − Indirect business taxes − Corporate income taxes − Undistributed corporate profits + Transfer payments

Personal disposable income = personal income – personal taxes

# Economics

Aggregate Income = Aggregate Expenditure

$C + S + T = C + I + G + (X - M)$

$S = I + (G - T) + (X - M)$ $\quad G - T = (S - I) - (X - M)$ $\quad (S - I) = (G - T) + (X - M)$

Production function: $Y = A\,\boldsymbol{F}(L,K)$

Growth in potential GDP = Growth in technology + $W_L$ (Growth in labor) + Wc (Growth in capital)

$W_L$ and $W_C$ are the relative share of labor and capital in the national income

Growth in per capita potential GDP = Growth in technology + Wc (Growth in K/L ratio)

Labor productivity = Real GDP/Aggregate hours; $Y/L = A\boldsymbol{F}(1, K/L)$

Potential GDP = Aggregate hours worked x Labor productivity

Potential GDP growth rate = Long-term growth rate of labor force + Long-term labor productivity growth rate

# Economics

Fractional reserve system: Money Created = New deposit / Reserve requirement

Money Multiplier = 1 / Reserve requirement

Quantity theory of money: $MV = PY$

Fischer effect: $R_{nom} = R_{real} + \prod e$

Fiscal multiplier = $1/[1 - c(1 - t)]$

$R_{P/B} = S_{P/B} \times P_B / P_P$

$F_{P/B} = S_{P/B} (1 + i_P) / (1+i_B)$

If $\omega_X \varepsilon_X + \omega_M (\varepsilon_M - 1) > 0$, a currency depreciation will reduce the trade deficit.

# FRA: Accounting

Assets = Liability + Equity

Equity = Contributed Capital + Retained Earnings

Assets = Liability + CC + BRE + Rev – Exp – Div

Profit = Revenue - Expenses

Comprehensive Income = Net Income + OCI

Revenue recognition, Percentage of completion method

Installment method: Profit = Cash * Expected Profit as % of Sales

$$\text{Basic EPS} = \frac{\text{Net income} - \text{Preferred dividends}}{\text{Weighted average number of shares outstanding}}$$

$$\text{Diluted EPS} = \frac{(\text{Net income})}{(\text{Weighted average number of shares outstanding} + \text{New common shares that would have been issued at conversion})}$$

# FRA: Cash Flow

Calculating CFO items: use the +/- technique

Cash Paid for New Equipment = Ending Gross Equipment Balance + Gross Cost of Equipment Sold: BB Equipment +Equip. Purchased - EB Equipment - Beginning Gross Equipment Balance

Cash from Sale = Historical Cost of Equipment Sold: BB Equipment + Equip. Purchased - EB Equipment - Depreciation on Equipment Sold: BB Acc. Depreciation + Dep. Expense - EB Acc. Depreciation + Gain on Sale of Equipment

FCFF = NI + NCC + Int(1-Tax rate) – FCInv – WCInv

FCFF = CFO + Int(1-Tax rate) – FCInv

FCFE = CFO – FCInv + Net borrowing

FCFE = CFO – FCInv – Net debt repayment

# FRA: Ratios

| Category | Measures | Example |
|---|---|---|
| **Activity ratios** | Efficiency | Revenue / Assets |
| **Liquidity ratios** | Ability to meet its short term obligations | Current Assets / Current Liabilities |
| **Solvency ratios** | Ability to meet long term debt obligations | Assets / Equity |
| **Profitability ratios** | Profitability | Net Income / Assets |
| **Valuation ratios** | Quantity of an asset or flow per share | Earnings / Number of Shares |

1) Name tells you balance sheet item
2) Balance sheet item → income statement item
3) Income statement item in the numerator
4) Average value of balance sheet number in denominator

DuPont:
ROE = NI/Assets x Assets/Equity
= NI/Revenue x Rev/Assets x Assets/Equity

| Activity Ratios | Numerator / Dominator |
|---|---|
| **Inventory turnover** | Cost of good sold / Average inventory |
| **Days of inventory on hand** | Number of days in period / Inventory turnover |

Cash conversion cycle (net operating cycle) = Days of inventory on hand + days of sales outstanding – number of days of payables

# FRA: Inventory, LLA, DTL, Bonds

FIFO and LIFO: use the 1 1 2 2 technique

WAC = Total cost of units available for sale / Total units available for sale

FIFO Inventory = LIFO Inventory + LIFO Reserve

FIFO COGS = LIFO COGS – (ending LIFO reserve – beginning LIFO reserve)

Carrying amount = historical cost – accumulated depreciation

Under IFRS: Impairment loss = Carrying Value – Recoverable amount

DTL = (Carrying Amount - Tax Base) x Tax Rate

ITE = ITP + Change in DTL – Change in DTA

Carrying amount of bond

# CF

**Capital Budgeting**

NPV and IRR formulas  Profitability index = PV for future cash flows / investment

AAR = Average net income/ average book value

**Cost of Capital**

$WACC = w_d r_d (1-t) + w_p r_p + w_e r_e$

YTM for cost of debt (IRR)  Cost of preferred stock = preferred dividend / share price

$r_e = Rf + \beta [E(R_{mkt}) - Rf]$  $r_e = R_f + \beta[E(r_{mkt}) - R_f + CRP]$

$P_0 = D_1 / (r_e - g)$ and $r_e = D_1 / P_0 + g$

**Breakpoint** = amount of capital at which the component cost of capital changes / weight of the component in the capital structure

$\beta_{asset} = \beta_{equity} \{1/1+[(1-t) D/E]\}$ and $\beta_{equity} = \beta_{asset} \{1+[(1-t) D/E]\}$

# CF

## Measures of Leverage

DOL = $\frac{\text{\% change in operating income}}{\text{\% change in sales}} = \frac{Q(P-V)}{Q(P-V)-F}$

DFL = $\frac{\text{\% change in net income}}{\text{\% change in operating income}} = \frac{[Q(P-V)-F]}{[Q(P-V)-F-C]}$

DTL = $\frac{\text{\% change in net income}}{\text{\% change in sales}} = \frac{Q(P-V)}{Q(P-V)-F-C}$

$Q_{BE}$ = [F + C] / [P – V]

$Q_{OBE}$ = F / [P – V]

## Dividends and Share Repurchases

If earnings yield (E/P) > after-tax yield on funds then EPS will increase, else EPS will decrease

If earnings yield (E/P) > after-tax yield cost of funds then EPS will increase, else EPS will decrease

Stock Price > Original BVPS → BVPS down

Stock Price < Original BVPS → BVPS up

# CF

| Ratio | Numerator | Denominator |
|---|---|---|
| Current ratio | Current assets | Current liabilities |
| Quick ratio | Cash + M/S + A/R | Current liabilities |
| Receivable turnover | Credit sales | Average receivables |
| Days of receivables | 365 | Receivable turnover |
| Inventory turnover | Cost of goods sold | Average inventory |
| Number of days of inventory | 365 | Inventory turnover |
| Payables turnover | Purchases | Average payables |
| Days of payables | 365 | Payables turnover |

**Operating cycle** = days of inventory + days of receivables

**Cash conversion cycle** = **Net operating cycle** =
average days of receivables + average days of inventory - average days of payables

| Yield | Formula |
|---|---|
| **Discount basis yield** | (F – P) / F x (360/T) |
| **Money market yield** | (F – P) / P x (360/T) |
| **BEY** | (F – P) / P x (365/T) |

$$\text{Cost of trade credit} = \left(1 + \frac{\text{Discount}}{1 - \text{Discount}}\right)^{\left(365 \,/\, \text{Number of days beyond discount period}\right)} - 1$$

Line of credit:

$$\text{Cost} = \frac{\text{Interest} + \text{Commitment fee}}{\text{Loan amount}}$$

Banker's Acceptance:

$$\text{Cost} = \frac{\text{Interest}}{\text{Net proceeds}} = \frac{\text{Interest}}{\text{Loan amount} - \text{Interest}}$$

Commercial Paper:

$$\text{Cost} = \frac{\text{Interest} + \text{Dealer's commission} + \text{Backup costs}}{\text{Loan amount} - \text{Interest}}$$

# PM

Diversification ratio = *Risk of equally weighted portfolio of n securities / Risk of single security selected at random*

$\rho\ (R_i, R_j) = Cov(R_i, R_j) / \sigma\ (R_i)\ \sigma\ (R_j)$

**Standard deviation: use the calculator**

$E(R_P) = w_1R_1 + w_2R_2$ $\sigma^2(R_P) = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\ \boldsymbol{\rho}\ \sigma_1\ \sigma_2$

Utility of an investment $= E(r) - ½\ A * \sigma^2$

CML Formula:

$$E(R_p) = R_f + \left(\frac{E(R_m) - R_f}{\sigma_m}\right) \times \sigma_p$$

Market Model: $R_i = \alpha_i + \beta R_m + e_i$ CAPM: $r_e = R_f + \beta\ [E(R_{mkt}) - R_f]$

Beta = Covariance of return on i and the market / Variance of the market return

Sharpe Ratio = $(R_P - R_f) / \sigma_P$ Treynor Ratio = $(R_P - R_f) / \beta_P$

$M^2 = (R_P - R_f)\ \sigma_m / \sigma_P - (R_m - R_f)$ Jensen's Alpha $\alpha_P = R_P - [R_f + \beta(R_m - R_f)]$

# Equity

Leverage ratio = A / E

*Margin Call Price = P x (1 – IM) / (1 – MM)*

Return = (cash at end / cash invested) – 1

ROE = NI / Avg Book Value of Equity

Gordon growth model: $V_0 = D_1 / (r- g)$
where g = growth rate = retention rate x return on equity

$P_0/E_1 = D_1 / E_1 / (r – g)$

EV = MVE + MVD + MVP – Cash and Cash Equivalents

# FIS

Pricing a bond with YTM

Pricing bonds with spot rates

Full price = Flat price + Accrued Interest

Accrued Interest = t / T

$$DR = \left(\frac{\text{Year}}{\text{Days}}\right) \times \left(\frac{FV - PV}{FV}\right)$$

$$AOR = \left(\frac{\text{Year}}{\text{Days}}\right) \times \left(\frac{FV - PV}{PV}\right)$$

$$PV = \frac{\frac{(\text{Index} + QM) \times FV}{m}}{\left(1 + \frac{\text{Index} + DM}{m}\right)^{1}} + \frac{\frac{(\text{Index} + QM) \times FV}{m}}{\left(1 + \frac{\text{Index} + DM}{m}\right)^{2}} + \cdots + \frac{\frac{(\text{Index} + QM) \times FV}{m} + FV}{\left(1 + \frac{\text{Index} + DM}{m}\right)^{N}}$$

$PV$ = present value, or the price of the floating-rate note
Index = reference rate, stated as an annual percentage rate
$QM$ = quoted margin, stated as an annual percentage rate
$FV$ = future value paid at maturity, or the par value of the bond
$m$ = periodicity of the floating-rate note, the number of payment periods per year
$DM$ = discount margin, the required margin stated as an annual percentage rate
$N$ = number of evenly spaced periods to maturity

# FIS

$$\text{MacDur} = \left\{\frac{1+r}{r} - \frac{1+r+\left[N\times(c-r)\right]}{c\times\left[(1+r)^{N}-1\right]+r}\right\} - (t/T)$$

$$\text{ModDur} = \frac{\text{MacDur}}{1+r}$$

$$\%\Delta PV^{Full} \approx -\text{AnnModDur} \times \Delta\text{yield}$$

$$\text{ApproxModDur} = \frac{(PV_{-}) - (PV_{+})}{2\times(\Delta\text{Yield})\times(PV_0)}$$

$$\text{EffDur} = \frac{(PV_{-}) - (PV_{+})}{2\times(\Delta\text{Curve})\times(PV_0)}$$

$$\text{MoneyDur} = \text{AnnModDur} \times PV^{Full}$$

$$\Delta PV^{Full} \approx -\text{MoneyDur}\times\Delta\text{Yield}$$

$$\text{PVBP} = \frac{(PV_{-}) - (PV_{+})}{2}$$

$$\text{ApproxCon} = \frac{(PV_{-}) + (PV_{+}) - \left[2\times(PV_0)\right]}{(\Delta\text{Yield})^2\times(PV_0)}$$

$$\text{EffCon} = \frac{\left[(PV_{-}) + (PV_{+})\right] - \left[2\times(PV_0)\right]}{(\Delta\text{Curve})^2\times(PV_0)}$$

$$\%\Delta PV^{Full} \approx (-\text{AnnModDur}\times\Delta\text{Yield}) + \left[\frac{1}{2}\times\text{AnnConvexity}\times(\Delta\text{Yield})^2\right]$$

# FIS

Single month mortality (SMM) measures prepayments in a month

SMM = Prepayment for month / (Beginning mortgage balance for month – Scheduled principal repayment for month)

The conditional prepayment rate (CPR) is an annualized version of SMM

A CPR of 6%, for example, means that approximately 6% of the outstanding mortgage balance at the beginning of the year is expected to be prepaid by the end of the year.

The 100 PSA prepayment benchmark is expressed as a monthly series of CPRs.

A PSA assumption greater than 100 PSA means that prepayments are assumed to be faster than the benchmark. In contrast, a PSA assumption lower than 100 PSA means that prepayments are assumed to be slower than the benchmark.

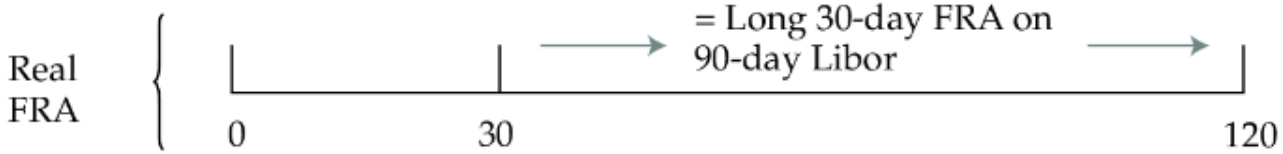
# Derivatives

$$F_0(T) = S_0(1+r)^T$$

$$F_0(T) = (S_0 - \gamma + \theta)(1+r)^T$$

$$V_T(T) = S_T - F_0(T)$$

$$V_t(T) = S_t - (\gamma - \theta)(1+r)^t - F_0(T)(1+r)^{-(T-t)}$$

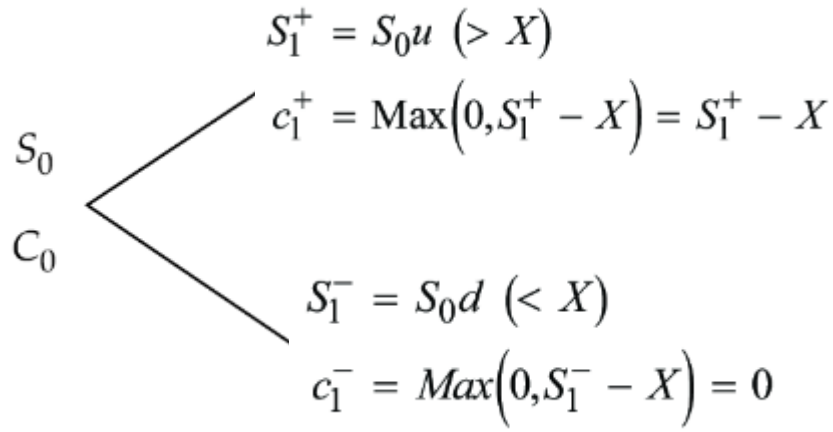



$$c_0 \geq \text{Max}\left[0, S_0 - X/(1+r)^T\right] \qquad p_0 \geq \text{Max}\left[0, X/(1+r)^T - S_0\right]$$

$$C_0 \geq \text{Max}\left[0, S_0 - X/(1+r)^T\right] \qquad P_0 \geq \text{Max}(0, X - S_0)$$

$$S_0 + p_0 = c_0 + X/(1+r)^T$$

$$F_0(T)/(1+r)^T + p_0 = c_0 + X/(1+r)^T$$




$$u = \frac{S_1^+}{S_0}, \ d = \frac{S_1^-}{S_0} \qquad c_0 = \frac{\pi c_1^+ + (1-\pi)c_1^-}{1+r}$$

where

$$\pi = \frac{1+r-d}{u-d}$$

# Derivatives

**Long Call: Payoff = Max (0, $S_T - X$)**
Derive max profit, max loss, breakeven from diagram

**Long Put: Payoff: Max (0, $X - S_T$)**
Derive max profit, max loss, breakeven from diagram

**Covered Call:**
Maximum profit = $X - S_0 + c_0$ Maximum **loss** = $S_0 - c_0$ Breakeven = $S_0 - c_0$

**Protective Put:**
Maximum profit = infinite Maximum **loss** = $S_0 + p_0 - X$ Breakeven = $S_0 + p_0$

# Alternative Investments

Hedge fund fee calculation

**Income based** REIT valuation:
FFO = Net Income + Depreciation – gains from sales of real estate + losses on sales of real estate

NAV = (MV of Total Assets – Total Liabilities)/ # of Shares

Future Price ≈ Spot Price (1+r) + Storage Costs – Convenience Yield

Futures price > spot price → contango

Futures price < spot price → backwardation

# Practice, Practice, Practice