Approved Prep Provider CFA Institute

2018 Level I Formulas

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Formula of Formulas

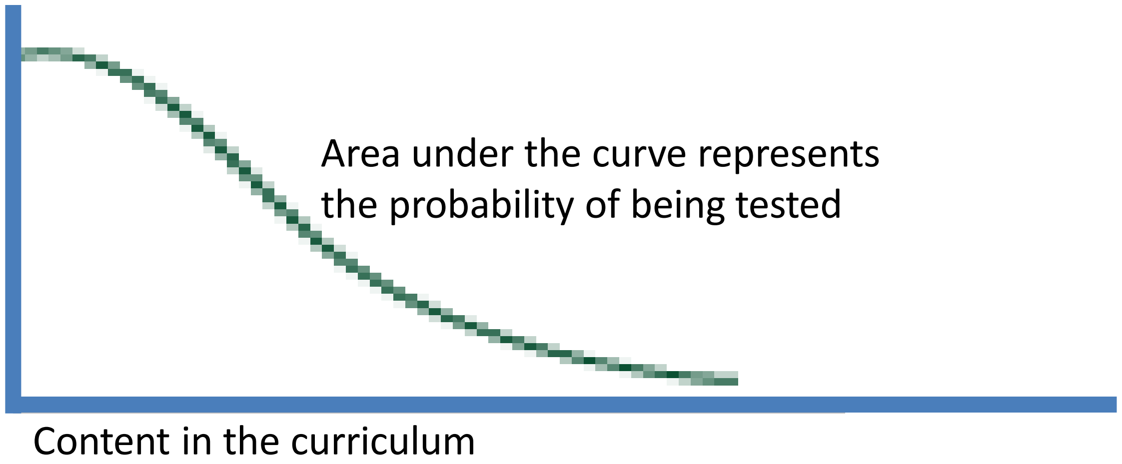
Have to know

Type 1: Formula exists, but what really matters is the intuition

Type 2: Know the formula, good to know the intuition

Should know

Type 3: Learn the formula, don't worry about the intuition



Nice to know

Type 4: Difficult formula and probability of being tested is low

Quant: TVM

Interest rate = Real risk-free rate + Inflation premium + Default risk premium + Liquidity premium + Maturity premium

$FV_N = PV (1 + r)^N$

$FV_N = PV\ e^{r N}$

EAR = (1 + Periodic interest rate)^m – 1

EAR = $e^r - 1$

Annuity formulas exist but use the calculator

PV of a perpetuity = A/r

Quant: DCF Applications

$NPV = \sum [CF_t / (1+r)^t]$

IRR is the rate which makes NPV = 0

Bank Discount Yield = $(D/F) \times 360/t$

Holding Period Yield = $(P_1 - P_0 + D) / P_0$

Money Market Yield = $HPY \times 360 / t$

Effective Annual Yield = $(1 + HPY)^{365/t} - 1$

Effective Annual Return = $(1 + \text{Periodic interest rate})^m – 1$

Quant: Statistics

Geometric Mean = $[(1+R_1)(1+R_2)\ldots\ldots(1+R_n)]^{1/n} - 1$

Harmonic Mean = $n / \sum (1/X_i)$

Weighted Mean = $\sum w_i X_i$

Location of observation at yth percentile: $L_y = (n + 1)(y/100)$

Population and sample variance: use the calculator

MAD = average of the absolute values of deviations from the mean

Range = maximum value – minimum value

Chebyshev's inequality states that for any set of observations, the proportion of the observations within k standard deviations of the mean is at least: $1 - (1/k^2)$ for all $k > 1$

Coefficient of variation = Risk / Return

Sharpe ratio = Excess return / Risk

Excess Kurtosis = Sample Kurtosis - 3

Quant: Probability

Multiplication rule: $P(AB) = P(A|B) \times P(B)$

Addition rule: $P(A \text{ or } B) = P(A) + P(B) - P(AB)$

Total probability rule: $P(A) = P(AS) + P(AS^C) = P(A|S)\,P(S) + P(A|S^C)\,P(S^C)$

$P(E \mid I) = P(E) \times P(I|E) / P(I)$

$Cov(R_i, R_j) = E[(R_i - ER_i)(R_j - ER_j)]$

$\rho\,(R_i, R_j) = Cov(R_i, R_j) / \sigma(R_i)\,\sigma(R_j)$

$E(R_P) = w_1R_1 + w_2R_2$

$\sigma^2(R_P) = w_1^2\sigma_1^2(R_1) + w_2^2\sigma_2^2(R_2) + 2w_1w_2Cov(R_1R_2)$

${}_nP_r = n! / (n - r)!$

${}_nC_r = n! / (n - r)!r!$

Quant: Distributions, Estimation, Hypothesis Testing

Binomial random variable: $p(x) = P(X = x) = {}_nC_x\, p^x (1 - p)^{n-x}$

Expected value = np and variance = n p (1 – p)

Normal distribution to standard normal: $z = (X - \mu) / \sigma$

$SFRatio = [E(R_p) - R_L] / \sigma_p$

Standard error of sample mean = $\sigma_X = \sigma / \sqrt{n}$ or $s_X = s / \sqrt{n}$

Confidence Interval = $X \pm z_{\alpha/2}(\sigma / \sqrt{n})$

Test statistic when testing for population mean: $\frac{\bar{X}_{RP} - \mu_0}{s_{\bar{X}}}$

Economics

Demand function
Inverse demand function

$$Q_A = 2 - 0.4\,P_A + 0.0005\,I + 0.10\,P_B - 0.15\,P_C$$

Supply function and inverse demand function

Consumer surplus

Producer surplus

Total surplus

Elasticity = % change in quantity demanded / % change in price

Flatter curve: more elastic

Elasticity of Demand = $\%\Delta Q / \%\Delta P = (\Delta Q / \Delta P) \times P/Q$

- Own price
- Substitute
- Complement
- Income

Top left: more elastic

Economics

Economic profit = Accounting profit – Total implicit opportunity costs

Economic profit = Total revenue – Total economic costs

Profit is maximized when MR = MC

In perfectly competitive markets:
P = MR = AR = D

Quantity, Price, Marginal Revenue

- $Q = 50 - 2P$
- $P = 25 - 0.5\,Q$
- $TR = PQ = 25\,Q - 0.5\,Q^2$
- $MR = 25 - Q_P$

In monopolistic markets:
MR = P [1 – 1/E]
Profit maximization condition: MR = MC
MC = P [1 – 1/E]
Profit maximizing price = MC / [1 – 1/E]

Economics

Aggregate Expenditure = Aggregate Output = Aggregate Income

GDP Deflator = (Nominal GDP / Real GDP) x 100

GDP based on expenditure approach = Consumer spending on goods and services + Business gross fixed investment + Change in inventories + Government spending on goods and services + Government gross fixed investment + Exports − Imports + Statistical discrepancy

GDP based on income approach = National income + Capital consumption allowance + Statistical discrepancy

National income = Compensation of employees + Corporate profits before taxes + Interest income + Unincorporated business net income + Rent + Indirect business taxes less subsidies

Personal income = National income − Indirect business taxes − Corporate income taxes − Undistributed corporate profits + Transfer payments

Personal disposable income = personal income – personal taxes

Economics

Aggregate Income = Aggregate Expenditure

$C + S + T = C + I + G + (X - M)$

$S = I + (G - T) + (X - M)$ $G - T = (S - I) - (X - M)$ $(S - I) = (G - T) + (X - M)$

Production function: $Y = A\,\boldsymbol{F}(L,K)$

Growth in potential GDP = Growth in technology + W_L (Growth in labor) + Wc (Growth in capital)

W_L and W_C are the relative share of labor and capital in the national income

Growth in per capita potential GDP = Growth in technology + Wc (Growth in K/L ratio)

Labor productivity = Real GDP/Aggregate hours; $Y/L = A\boldsymbol{F}(1, K/L)$

Potential GDP = Aggregate hours worked x Labor productivity

Potential GDP growth rate = Long-term growth rate of labor force + Long-term labor productivity growth rate

Economics

Fractional reserve system: Money Created = New deposit / Reserve requirement

Money Multiplier = 1 / Reserve requirement

Quantity theory of money: $MV = PY$

Fischer effect: $R_{nom} = R_{real} + \prod e$

Fiscal multiplier = $1/[1 – c(1 – t)]$

$R_{P/B} = S_{P/B} \times P_B / P_P$

$F_{P/B} = S_{P/B} (1 + i_P) / (1+i_B)$

If $\omega_X \varepsilon_X + \omega_M (\varepsilon_M - 1) > 0$, a currency depreciation will reduce the trade deficit.

FRA: Accounting

Assets = Liability + Equity

Equity = Contributed Capital + Retained Earnings

Assets = Liability + CC + BRE + Rev – Exp – Div

Profit = Revenue - Expenses

Comprehensive Income = Net Income + OCI

Revenue recognition, Percentage of completion method

Installment method: Profit = Cash * Expected Profit as % of Sales

$$\text{Basic EPS} = \frac{\text{Net income} - \text{Preferred dividends}}{\text{Weighted average number of shares outstanding}}$$

$$\text{Diluted EPS} = \frac{(\text{Net income})}{(\text{Weighted average number of shares outstanding} + \text{New common shares that would have been issued at conversion})}$$

FRA: Cash Flow

Calculating CFO items: use the +/- technique

Cash Paid for New Equipment = Ending Gross Equipment Balance + Gross Cost of Equipment Sold: BB Equipment +Equip. Purchased - EB Equipment - Beginning Gross Equipment Balance

Cash from Sale = Historical Cost of Equipment Sold: BB Equipment + Equip. Purchased - EB Equipment - Depreciation on Equipment Sold: BB Acc. Depreciation + Dep. Expense - EB Acc. Depreciation + Gain on Sale of Equipment

FCFF = NI + NCC + Int(1-Tax rate) – FCInv – WCInv

FCFF = CFO + Int(1-Tax rate) – FCInv

FCFE = CFO – FCInv + Net borrowing

FCFE = CFO – FCInv – Net debt repayment

FRA: Ratios

Category	Measures	Example
Activity ratios	Efficiency	Revenue / Assets
Liquidity ratios	Ability to meet its short term obligations	Current Assets / Current Liabilities
Solvency ratios	Ability to meet long term debt obligations	Assets / Equity
Profitability ratios	Profitability	Net Income / Assets
Valuation ratios	Quantity of an asset or flow per share	Earnings / Number of Shares

1) Name tells you balance sheet item
2) Balance sheet item → income statement item
3) Income statement item in the numerator
4) Average value of balance sheet number in denominator

Activity Ratios	Numerator / Dominator
Inventory turnover	Cost of good sold / Average inventory
Days of inventory on hand	Number of days in period / Inventory turnover

DuPont:
ROE = NI/Assets x Assets/Equity
= NI/Revenue x Rev/Assets x Assets/Equity

Cash conversion cycle (net operating cycle) = Days of inventory on hand + days of sales outstanding – number of days of payables

FRA: Inventory, LLA, DTL, Bonds

FIFO and LIFO: use the 1 1 2 2 technique

WAC = Total cost of units available for sale / Total units available for sale

FIFO Inventory = LIFO Inventory + LIFO Reserve

FIFO COGS = LIFO COGS – (ending LIFO reserve – beginning LIFO reserve)

Carrying amount = historical cost – accumulated depreciation

Under IFRS: Impairment loss = Carrying Value – Recoverable amount

DTL = (Carrying Amount - Tax Base) x Tax Rate

ITE = ITP + Change in DTL – Change in DTA

Carrying amount of bond

CF

Capital Budgeting

NPV and IRR formulas

Profitability index = PV for future cash flows / investment

AAR = Average net income/ average book value

Cost of Capital

$WACC = w_d r_d (1-t) + w_p r_p + w_e r_e$

YTM for cost of debt (IRR)

Cost of preferred stock = preferred dividend / share price

$r_e = Rf + \beta [E(R_{mkt}) – Rf]$ $r_e = R_f + \beta[E(r_{mkt}) – R_f + CRP]$

$P_0 = D_1 / (r_e - g)$ and $r_e = D_1 / P_0 + g$

Breakpoint = amount of capital at which the component cost of capital changes / weight of the component in the capital structure

$\beta_{asset} = \beta_{equity} \{1/1+[(1-t) D/E]\}$ and $\beta_{equity} = \beta_{asset} \{1+[(1-t) D/E]\}$

CF

Measures of Leverage

DOL = % change in operating income / % change in sales $= \frac{Q(P-V)}{Q(P-V)-F}$

DFL = % change in net income / % change in operating income $= \frac{[Q(P-V)-F]}{[Q(P-V)-F-C]}$

DTL = % change in net income / % change in sales $= \frac{Q(P-V)}{Q(P-V)-F-C}$

Q_{BE} = [F + C] / [P – V]

Q_{OBE} = F / [P – V]

CF

Ratio	Numerator	Denominator
Current ratio	Current assets	Current liabilities
Quick ratio	Cash + M/S + A/R	Current liabilities
Receivable turnover	Credit sales	Average receivables
Days of receivables	365	Receivable turnover
Inventory turnover	Cost of goods sold	Average inventory
Number of days of inventory	365	Inventory turnover
Payables turnover	Purchases	Average payables
Days of payables	365	Payables turnover

Operating cycle = days of inventory + days of receivables

Cash conversion cycle = **Net operating cycle** =
average days of receivables + average days of inventory - average days of payables

Yield	Formula
Discount basis yield	(F – P) / F x (360/T)
Money market yield	(F – P) / P x (360/T)
BEY	(F – P) / P x (365/T)

$$\text{Cost of trade credit} = \left(1 + \frac{\text{Discount}}{1 - \text{Discount}}\right)^{\left(365 \Big/ \substack{\text{Number of days} \\ \text{beyond discount period}}\right)} - 1$$

Line of credit:

$$\text{Cost} = \frac{\text{Interest} + \text{Commitment fee}}{\text{Loan amount}}$$

Banker's Acceptance:

$$\text{Cost} = \frac{\text{Interest}}{\text{Net proceeds}} = \frac{\text{Interest}}{\text{Loan amount} - \text{Interest}}$$

Commercial Paper:

$$\text{Cost} = \frac{\text{Interest} + \text{Dealer's commission} + \text{Backup costs}}{\text{Loan amount} - \text{Interest}}$$

PM

Diversification ratio = *Risk of equally weighted portfolio of n securities / Risk of single security selected at random*

ρ (R_i, R_j) = Cov(R_i, R_j) / σ (R_i) σ (R_j)

Standard deviation: use the calculator

$E(R_P) = w_1R_1 + w_2R_2$ $\sigma^2(R_P) = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2 \boldsymbol{\rho} \sigma_1 \sigma_2$

Utility of an investment = $E(r) - ½ A * \sigma^2$

CML Formula:

$$E(R_p) = R_f + \left(\frac{E(R_m) - R_f}{\sigma_m}\right) \times \sigma_p$$

Market Model: $R_i = \alpha_i + \beta R_m + e_i$ CAPM: $r_e = R_f + \beta\,[E(R_{mkt}) - R_f]$

Beta = Covariance of return on i and the market / Variance of the market return

Sharpe Ratio = $(R_P - R_f) / \sigma_P$ Treynor Ratio = $(R_P - R_f) / \beta_P$

$M^2 = (R_P - R_f)\,\sigma_m / \sigma_P - (R_m - R_f)$ Jensen's Alpha $\alpha_P = R_P - [R_f + \beta(R_m - R_f)]$

Equity

Leverage ratio = A / E

Margin Call Price = P x (1 – IM) / (1 – MM)

Return = (cash at end / cash invested) – 1

ROE = NI / Avg Book Value of Equity

Gordon growth model: $V_0 = D_1 / (r - g)$
where g = growth rate = retention rate x return on equity

$P_0 / E_1 = D_1 / E_1 / (r - g)$

EV = MVE + MVD + MVP – Cash and Cash Equivalents

FIS

Pricing a bond with YTM

Pricing bonds with spot rates

Full price = Flat price + Accrued Interest

Accrued Interest = t / T

$$DR = \left(\frac{\text{Year}}{\text{Days}}\right) \times \left(\frac{FV - PV}{FV}\right)$$

$$AOR = \left(\frac{\text{Year}}{\text{Days}}\right) \times \left(\frac{FV - PV}{PV}\right)$$

$$PV = \frac{\frac{(\text{Index} + QM) \times FV}{m}}{\left(1 + \frac{\text{Index} + DM}{m}\right)^1} + \frac{\frac{(\text{Index} + QM) \times FV}{m}}{\left(1 + \frac{\text{Index} + DM}{m}\right)^2} + \cdots + \frac{\frac{(\text{Index} + QM) \times FV}{m} + FV}{\left(1 + \frac{\text{Index} + DM}{m}\right)^N}$$

PV = present value, or the price of the floating-rate note

Index = reference rate, stated as an annual percentage rate

QM = quoted margin, stated as an annual percentage rate

FV = future value paid at maturity, or the par value of the bond

m = periodicity of the floating-rate note, the number of payment periods per year

DM = discount margin, the required margin stated as an annual percentage rate

N = number of evenly spaced periods to maturity

FIS

$$\text{MacDur} = \left\{ \frac{1+r}{r} - \frac{1+r+[N \times (c-r)]}{c \times [(1+r)^N - 1] + r} \right\} - (t/T)$$

$$\text{ModDur} = \frac{\text{MacDur}}{1+r}$$

$$\%\Delta PV^{Full} \approx -\text{AnnModDur} \times \Delta\text{yield}$$

$$\text{ApproxModDur} = \frac{(PV_-) - (PV_+)}{2 \times (\Delta\text{Yield}) \times (PV_0)}$$

$$\text{EffDur} = \frac{(PV_-) - (PV_+)}{2 \times (\Delta\text{Curve}) \times (PV_0)}$$

$$\text{MoneyDur} = \text{AnnModDur} \times PV^{Full}$$

$$\Delta PV^{Full} \approx -\text{MoneyDur} \times \Delta\text{Yield}$$

$$\textbf{PVBP} = \frac{(PV_-) - (PV_+)}{2}$$

$$\text{ApproxCon} = \frac{(PV_-) + (PV_+) - [2 \times (PV_0)]}{(\Delta\text{Yield})^2 \times (PV_0)}$$

$$\text{EffCon} = \frac{[(PV_-) + (PV_+)] - [2 \times (PV_0)]}{(\Delta\text{Curve})^2 \times (PV_0)}$$

$$\%\Delta PV^{Full} \approx$$
$$(-\text{AnnModDur} \times \Delta\text{Yield}) + \left[\frac{1}{2} \times \text{AnnConvexity} \times (\Delta\text{Yield})^2\right]$$

FIS

Single month mortality (SMM) measures prepayments in a month

SMM = Prepayment for month / (Beginning mortgage balance for month – Scheduled principal repayment for month)

The conditional prepayment rate (CPR) is an annualized version of SMM

A CPR of 6%, for example, means that approximately 6% of the outstanding mortgage balance at the beginning of the year is expected to be prepaid by the end of the year.

The 100 PSA prepayment benchmark is expressed as a monthly series of CPRs.

A PSA assumption greater than 100 PSA means that prepayments are assumed to be faster than the benchmark. In contrast, a PSA assumption lower than 100 PSA means that prepayments are assumed to be slower than the benchmark.

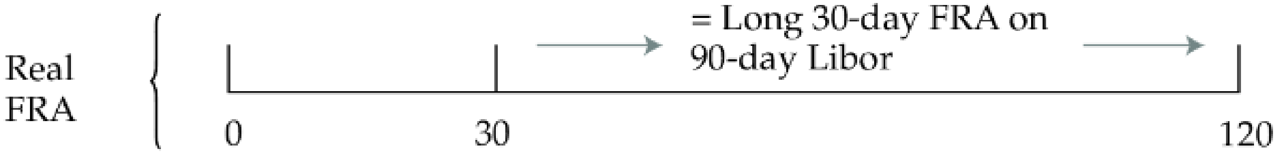
Derivatives

$$F_0(T) = S_0(1+r)^T$$

$$F_0(T) = (S_0 - \gamma + \theta)(1+r)^T$$

$$V_T(T) = S_T - F_0(T)$$

$$V_t(T) = S_t - (\gamma - \theta)(1+r)^t - F_0(T)(1+r)^{-(T-t)}$$



$$c_0 \geq \text{Max}\left[0, S_0 - X/(1+r)^T\right] \qquad p_0 \geq \text{Max}\left[0, X/(1+r)^T - S_0\right]$$

$$C_0 \geq \text{Max}\left[0, S_0 - X/(1+r)^T\right] \qquad P_0 \geq \text{Max}(0, X - S_0)$$

$$S_0 + p_0 = c_0 + X/(1+r)^T$$

$$F_0(T)/(1+r)^T + p_0 = c_0 + X/(1+r)^T$$

S_0, C_0

$$S_1^+ = S_0 u \; (> X)$$

$$c_1^+ = \text{Max}\left(0, S_1^+ - X\right) = S_1^+ - X$$

$$S_1^- = S_0 d \; (< X)$$

$$c_1^- = Max\left(0, S_1^- - X\right) = 0$$

$$u = \frac{S_1^+}{S_0}, \quad d = \frac{S_1^-}{S_0}$$

$$c_0 = \frac{\pi c_1^+ + (1-\pi)c_1^-}{1+r}$$

where

$$\pi = \frac{1+r-d}{u-d}$$

Alternative Investments

Hedge fund fee calculation

Income based REIT valuation:
FFO = Net Income + Depreciation – gains from sales of real estate + losses on sales of real estate

NAV = (MV of Total Assets – Total Liabilities)/ # of Shares

Future Price ≈ Spot Price (1+r) + Storage Costs – Convenience Yield

Futures price > spot price → contango

Futures price < spot price → backwardation

Practice, Practice, Practice