

2018 CFA® Exam Prep

IFT Study Notes

Volume 1

Ethical and Professional Standards

Quantitative Methods

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R01 Ethics and Trust in the Investment Profession

1. Introduction

To illustrate the importance of ethical behavior, the curriculum cites the example of an analyst's action at a financial services firm. The research department at the firm is responsible for making investment recommendations to clients after sound analysis and valuation of companies. One of the analysts at the firm misrepresents facts in his report with the objective of pleasing the management of the subject company. He hoped this would lead to financial benefits for his employer and himself. Clients who acted on the recommendation incur heavy losses and spread a negative word on several online forums about the firm. This eventually affects the reputation of the firm, forcing it to downsize and many employees lose their job. This example illustrates how one member's unethical actions have a spiraling effect on the firm and other employees for no fault of theirs.

The foundation of the investment management industry is trust. The top three attributes of an investment management firm are as follows:

- Has transparent and open business practices.
- Takes responsible actions to address an issue or crisis.
- Has ethical business practices.

Ethical behavior is not just about adhering to the law, rules, and regulations. It is about identifying potential conflicts and acting righteously in situations where there are no stated rules.

2. Ethics

The word ethics comes from the Greek word "ethos" meaning character, guiding beliefs or ideals. There are several definitions of ethics all of which essentially convey the same meaning.

Ethics can be described as a set of moral principles and rules of conduct that provide guidance for our behavior. Ethical principles define what is good, acceptable behavior and what is forbidden or unacceptable behavior. Examples of ethical principles include honesty, diligence, justice, being open about the costs involved in an investment, fairness, and respect for the rights of others.

Another definition of ethical conduct is behavior that balances one's own interest with the direct and indirect consequences of the behavior on others.

Instructor's Note:

The 'others' are often referred to as stakeholders, i.e. groups of people or individuals who are directly or indirectly impacted by our decisions.

Specific communities formally define the rules for acceptable and forbidden behavior into a written set of principles called the **code of conduct**. Professional associations, universities

and companies often adopt a code of ethics and expect their members to adhere to those rules, at the very least. The members may choose to display higher standards of behavior than what is stipulated in the code of ethics.

Some communities may also expand on their code of ethics and adopt explicit rules or standards that identify specific behaviors required of community members. These **standards of conduct** serve as a benchmark of the minimally acceptable behavior expected of members of a community.

CFA Institute is an example of a community that has laid down a code of ethics and standards of conduct for its members and candidates to follow. The set of principles comprising the Code of Ethics and Standards of Professional Conduct is clearly documented in the CFA Institute *Standards of Practice Handbook*.

Members and candidates are required to pledge their commitment to abide by the Code and Standards each year. They are also required to disclose any violations of the Code and Standards in the Professional Conduct Statement each year. Members who violate the Code and Standards face disciplinary action.

3. Ethics and Professionalism

An occupation can be divided into: job, vocation, and profession. A job is a work one does to earn a livelihood, or earn money. A vocation is a job that one is passionate about doing; one derives a sense of satisfaction or meaning from it, as it is his/her *calling*. A profession is the ultimate evolution of occupation. It:

- 1) requires specialized training and skills,
- 2) is based on service to others, and
- 3) is practiced by members who share and adhere to a common code of ethics.

Professionals use their acquired skills to serve their clients. Clients differ from customers; a customer is one who engages in a single or a series of transactions to buy a good or service. This relationship is transactional in nature. A client, on the other hand, uses the services of a professional on an ongoing basis, for a fee. The basis of this relationship is trust and the client's interests take priority over personal or employer's interests.

In any given profession, the code of ethics openly communicates the established principles of the profession and how its members are expected to behave. It helps in building public confidence that members of the profession will use their skills and knowledge for the benefit of their clients.

4. Challenges to Ethical Conduct

Some of the challenges to ethical conduct include the following:

Overestimating one's morality: People believe they are more ethical than they actually are. This overconfidence in themselves can sometimes lead to faulty decision making. It is often seen that emotions cloud rational thinking, prompting one to make decisions that may not be

the most ethical choice.

Situational influences: These are external factors such as cultural, social and environmental factors that influence one's thinking, behavior, and decision making. Some of the common situational influences are:

- **Money and prestige:** Both money and prestige push people to act in their own self-interests and take actions that are less ethical. The promise of a large financial bonus or a promotion, can impact people's thinking ability and cause them to act in their own short term interests and ignore the long-term consequences of their actions.
- **Loyalty to employer, employee, and colleagues:** Loyalty can have both positive and negative effects. For instance, some colleagues may encourage you to behave more ethically and enroll in the CFA Program to advance your career. On the other hand, colleagues who do not adhere to the Code and Standards may encourage you to simply act in accordance with the local law, even though it may fall short of ethical conduct.
- **Compliance culture:** A strong compliance policy is important for ethical decision making; however, processes focused solely on compliance oversimplify decision making and do not help the larger cause. The curriculum cites the example of Enron, which engaged in transactions with third-party entities where Enron's CFO, Andrew Fastow had a personal interest. In keeping with the spirit of compliance, Fastow sought approvals from the board of directors for all the proposed transactions with Enron. The board failed to see beyond the compliance requirements and did not question Fastow's vested interests that were not aligned with that of Enron's shareholders.

5. The Importance of Ethical Conduct in the Investment Industry

The investment industry connects two parties: investors and borrowers. Borrowers are those who are in need of capital to fund their long-term goals or their regular operations. Long-term goals may include building schools, factories, bridges, etc. Investors are those who supply capital and seek a return. The investment industry bridges the gap between those who are in need of capital and those who are willing to provide capital.

The foundation of the investment industry is built on trust. All the participants in the system must act ethically to build an environment of trust. For instance, if investors trust their financial advisers and financial markets, in general, then they will be willing to lend capital, take risks, and not panic over price fluctuations in the short term. This will encourage more participation in the financial markets and capital flow to fund the growth of several projects that will largely benefit the society. Similarly, organizations in need of capital will be more willing to expand their businesses if they are reasonably assured of attracting funds.

While trust is important in any business, it is particularly important in the investment profession because of the following reasons:

- **Nature of the client relationship:** Investors park their assets with financial institutions because they trust the firms to safeguard their assets. If the firm and its employees breach this trust and fail to protect their client's assets, they will

eventually lose business as clients will no longer entrust the firm with their assets.

- **Difference in knowledge and access to information:** Investment managers have more access to information and more specialized knowledge than their clients. Clients trust the professionals will use the information and knowledge for the benefit of the client's interests and in no way will act to their disadvantage.
- **Nature of investment products and services:** Unlike other industries, the products and services in the investment industry are intangible. They cannot be touched or physically felt to judge their quality. In the investment industry, assets are often notional with values measured in the form of numbers. For instance, the value of the investments as presented by one's financial adviser are mere numbers printed electronically. Investors trust that the information presented to them is complete, accurate, and presents a fair picture.

6. Ethical vs. Legal Standards

There is a grey area between what is legally accepted and what is ethical. Acting in accordance with the law and acting ethically are not necessarily the same. There are four possible outcomes for any action from a legal and ethical perspective:

- Not legal but ethical: For example, civil disobedience or protesting peacefully against an issue may not be legal, but it is ethical. Another example of an illegal but ethical act is that of whistleblowing. Whistleblowing is raising the curtain off an illegal or corrupt activity.
- Not legal and not ethical.
- Legal and ethical.
- Legal but not necessarily ethical: Some countries do not have laws that prohibit trading while in possession of material nonpublic information. While this act of trading is legal from the local country's perspective, it is considered unethical by the CFA Institute and other investment professionals.

There are several reasons why laws are not sufficient to ensure ethical conduct among market participants, as discussed below:

- Laws and regulations are often created in response to existing market practices. A new law might address an existing ethical problem but create an opportunity for other unethical behavior in future.
- Laws can be interpreted differently. Market participants may choose to interpret the law to their advantage or delay compliance where there is no punitive action.
- Laws can vary across jurisdictions. This may encourage questionable practice to move to places that are less restrictive in nature.

Ethical conduct encourages us to:

- Go beyond what is legally required.
- Consider the impact on all stakeholders.

- Make good choices, even in the absence of clear laws and regulations.

7. Ethical Decision-Making Frameworks

Firms must strive to develop a strong ethical culture and encourage investment professionals to apply ethical decision-making skills every day; so much so that it becomes second nature. Working and operating in an environment that fosters integrity and motivates its employees to do the right thing will go a long way in preventing unethical actions.

Setting up an ethical framework reinforces investment professionals to do the right thing. The ethical framework:

- Helps in evaluating a situation from multiple perspectives after considering the larger picture in such a way that it benefits stakeholders in the long-term. Often, the impact of a decision or all aspects of a situation is not clear in the short-term and decisions taken in haste may harm stakeholders unintentionally.
- Helps decision makers justify actions to a broader group of stakeholders.

The following ethical decision making framework is presented in the curriculum.

- **Identification phase:** Identify all the relevant facts. This includes information one has and what one would like to have.
 - Identify relevant facts such as details of the employer, information on an IPO or a deal, rules and regulations of the industry etc.
 - Identify the stakeholders such as employer, market participants, clients, supervisor, investors, family etc.
 - Identify relevant ethical principles for the situation. This may include loyalty to employer, clients' interests take precedence before everything else, and maintaining the confidentiality of information.
 - Identify any potential conflicts of interest, or conflicts in your duties to employers/clients. Examples of potential conflict of interest include duties to one client versus other clients of the firm, financial rewards linked to the success of a deal versus duty to employer, and duty to supervisor versus the need to impress.
- **Consideration phase:** Seek guidance to navigate through situational influences and personal biases that may affect decision making.
 - Examples of situational influences include how much fees the firm will earn from a deal, how much bonus or compensation one expects to receive because of working on an IPO/deal, or associating one's self-worth to working on a prestigious account/deal.
 - Examples of where one could seek guidance include the firm's compliance department, peers, the CFA Institute Code and Standards or a supervisor.
- **Decide and act:** Make a decision and act.
- **Reflect:** Once the decision is made, assess the decision to see if it had the desired outcome. If not, then analyze the reasons: were the stakeholders identified, was there

any conflict of interest, were the ethical principles identified, did you seek guidance for how to deal with situational influences and personal behavioral biases?

Sometimes the information is not sufficient to make a decision in which case the process becomes iterative as you seek guidance to gather more relevant information.

Summary

LO.a: Explain ethics.

The word ethics is derived from the Greek word 'ethos,' which means character. Ethics means making good choices. Ethics includes a set of moral principles and rules of conduct that help us in our behavior.

LO.b: Describe the role of a code of ethics in defining a profession.

Profession is the final development of an occupation.

Profession is:

- based on specialized knowledge and skills.
- based on service to others.
- practiced by members who share and agree to adhere to a common code of ethics.

In any given profession, the code of ethics openly communicates the established principles of the profession and how its members are expected to behave.

It helps in building public confidence that members of the profession will use their skills and knowledge for the benefit of their clients.

LO.c: Identify challenges to ethical behavior.

One challenge is that people tend to believe that their ethical standards are above average. This leads to overconfidence bias and therefore people place too much importance on their internal traits.

However, studies show that external factors (situational influence) are the main determinant of ethical behavior. They shift our focus to the immediate rather than long-term impacts of a decision. The three main types of situational influences are:

- Money & Prestige.
- Loyalty to employer and/or colleagues.
- Strong compliance culture.

LO.d: Describe the need for high ethical standards in the investment industry.

High ethical standards are always important. However, they are of particular importance in the investment industry, because this industry is based almost entirely on trust. Also the products and services of this industry are intangible in nature.

Clients trust investment professionals to use their skills and knowledge for their benefit and to protect their assets.

If investment professionals adhere to high ethical standards, all stakeholders gain long-term benefits.

LO.e: Distinguish between ethical and legal standards.

Legal and ethical conduct is not always the same.

Laws are not always the best mechanism to reduce unethical behavior because:

- Legal standards are often created to address past ethical failings. They do not provide direction for an ever changing and increasingly complex world.
- Laws are often rule based.
- Laws will vary across countries.

Ethical conduct goes beyond legal standards.

LO.f: Describe and apply a framework for ethical decision making.

A framework for ethical decision making can help people look at and assess a decision from different perspectives. This enables them to make good decisions, and to limit unplanned consequences.

A general ethical decision making framework has the following four steps.

1. Identify: Relevant facts, stakeholders and duties owed, ethical principles, conflicts of interest.
2. Consider: Situational influences, additional guidance, alternative actions.
3. Decide and act.
4. Reflect: Was the outcome as anticipated? Why or why not?

Practice Questions

1. Which of the following statements is *most likely* correct?
 - A. Ethics can be described as a set of moral principles that provide guidance for our behaviour.
 - B. Ethical conduct is behaviour that balances one's own interest with only the direct consequences of the behaviour on others.
 - C. Professional associations adopt a code of ethics to protect their own professional community.
 2. Which one of the following is a *least likely* reason for a profession to establish a code of ethics?
 - A. A code of ethics serves as an aid in decision-making.
 - B. A code of ethics helps instill confidence among clients and prospective clients.
 - C. A code of ethics helps ensure that members of the profession will follow the law.
 3. Which of the following is *least likely* a challenge faced by professionals to display ethical behaviour?
 - A. People tend to believe they are more ethical than they actually are.
 - B. People tend to underestimate their own morality.
 - C. People tend to underestimate the impact of situational influences.
 4. Which of the following statements is *least likely* accurate? Trust is particularly important in the investment profession because:
 - A. investment professionals have specialized knowledge and access to information is asymmetrical.
 - B. products and services in the investment industry tend to be intangible.
 - C. returns cannot be guaranteed for most types of investments.
 5. Which of the following is *least likely* a reason for laws being insufficient to ensure ethical conduct among market participants?
 - A. Laws can be interpreted differently.
 - B. Laws are largely the same across jurisdictions.
 - C. Passing a law takes significant time.
 6. Which of the following statements about ethical decision framework is/are *most likely* accurate?
 - **Statement 1:** An ethical decision framework helps decision makers justify actions to stakeholders.
 - **Statement 2:** Too many choices can at best lead to inaction.
 - **Statement 3:** An ethical decision framework serves as a tool for investment
-

professionals to choose the best possible alternative.

- A. Statement 1 and 3.
- B. Statement 3 only.
- C. Statement 1, 2 and 3.

Solutions

1. A is correct. Statement B is incorrect because ethical conduct is behaviour that balances one's own interest with the direct and indirect consequences of the behaviour on others. Statement C is incorrect. Section 2. LO.a.
2. C is correct. The code of ethics cannot ensure that members of the profession will follow the law. Statements A and B are true. Section 3. LO.a.
3. B is correct. Challenges faced by professionals to display ethical behaviour include: 1) overestimating one's morality and 2) underestimating the effect of situational influences. Section 4. LO.c.
4. C is correct. Statement A and B are valid reasons for trust being important in the investment profession. C does not represent a reason for why trust is particularly important in the investment industry. Section 5. LO.d.
5. B is correct. Laws can vary across jurisdictions. This may encourage questionable practice to move to places that are less restrictive in nature. Statements A and C are valid reasons for why the law alone might be insufficient to ensure ethical behaviour. Section 6. LO.e.
6. C is correct. All three statements regarding ethical decision frameworks are correct. Section 7. LO.f.

R02 Code of Ethics and Standards of Professional Conduct Profession

Introduction

The Code of Ethics and Standards of Professional Conduct (Code and Standards) stipulate rules and guidelines for CFA Institute members and candidates on how to conduct themselves in a professional and ethical manner, with integrity, under all circumstances for the benefit of society.

CFA Institute Professional Conduct Program

All CFA Institute members and candidates enrolled in the CFA program are required to comply with the Code and Standards. The Professional Conduct Program (PCP), in conjunction with the Disciplinary Review Committee (DRC), is responsible for enforcement of the Code and Standards.

- The DRC is a volunteer committee of CFA charterholders who serve on panels to review conduct and partner with professional conduct staff to establish and review professional conduct policies.
- The CFA Institute Bylaws and Rules of Procedure for Professional Conduct form the basic structure for enforcing the Code and Standards.

Professional Conduct **inquiries** can be prompted by several reasons:

- Self-disclosure: On the annual Professional Conduct Statement, members disclose if they have been a subject of civil litigation or criminal investigation.
- Written complaints: The PCP staff may receive written complaints.
- Evidence of misconduct: The PCP staff may come across violations through media or any public source.
- Report by a CFA exam proctor: If a candidate violated any rules on exam day and is reported by the proctor.
- CFA Institute may conduct analyses of scores and exam materials after the exam, as well as monitor online and social media to detect disclosure of confidential exam information. For instance, if a friend sits for the exam in Australia and discusses specific questions online after the exam while you are in the United States where the exam hasn't started, it can be considered a violation of the Code and Standards.

After an inquiry is initiated, the professional conduct team conducts an **investigation**. If the professional conduct staff believes a violation of the Code and Standards has occurred, they propose sanctions which can include: public censure, suspension of membership and use of the CFA designation, and revocation of the CFA charter. Candidates may be suspended or prohibited from further participation in the CFA program.

The member or candidate has the opportunity to accept or reject any charges and the proposed sanctions. If the member/candidate accepts the violation, then the sanctions will

be imposed. If the member/candidate does not accept the charges or the proposed sanctions, the matter is referred to the **DRC**, which reviews materials and presentations from professional conduct staff and from the member or candidate. The panel's task is to determine whether a violation of the Code and Standards or testing policies has occurred and, if so, what sanction should be imposed.

Code of Ethics

Members of the CFA Institute (including CFA charter holders) and candidates for the CFA designation ("Members and candidates") must:

1. Act with integrity, competence, diligence, and respect and in an ethical manner with the public, clients, prospective clients, employers, employees, colleagues in the investment profession, and other participants in the global capital markets.
2. Place the integrity of the investment profession and interests of clients above their own personal interests.
3. Use reasonable care and exercise independent professional judgment when conducting investment analysis, making investment recommendations, taking investment actions, and engaging in other professional activities.
4. Practice and encourage others to practice in a professional and ethical manner that will reflect credit on themselves and the profession.
5. Promote the integrity and viability of the global capital markets for the ultimate benefit of society.
6. Maintain and improve their professional competence and strive to maintain and improve the competence of other investment professionals.

Instructor's Note:

The six components of Code of Ethics, outlined above, are important and should be memorized.

Standards of Professional Conduct

There are seven Standards of Professional Conduct. Each standard has sub-sections. The standards are covered in detail in the next reading.

- I. Professionalism
- II. Integrity of Capital Markets
- III. Duties to Clients
- IV. Duties to Employers
- V. Investment analysis, Recommendations, and Actions
- VI. Conflicts of Interest
- VII. Responsibilities as a CFA Institute Member, or CFA Candidate

I. Professionalism**A. Knowledge of the Law**

Members and Candidates must understand and comply with all applicable laws, rules, and regulations (including the CFA Institute Code of Ethics and Standards of Professional Conduct) of any government, regulatory organization, licensing agency, or professional association governing their professional activities. In the event of conflict, Members and Candidates must comply with the more strict law, rule, or regulation. Members and Candidates must not knowingly participate or assist in and must dissociate from any violation of such laws, rules, or regulations.

B. Independence and Objectivity

Members and Candidates must use reasonable care and judgment to achieve and maintain independence and objectivity in their professional activities. Members and Candidates must not offer, solicit, or accept any gift, benefit, compensation, or consideration that reasonably could be expected to compromise their own or another's independence and objectivity.

C. Misrepresentation

Members and Candidates must not knowingly make any misrepresentations relating to investment analysis, recommendations, actions, or other professional activities.

D. Misconduct

Members and Candidates must not engage in any professional conduct involving dishonesty, fraud, or deceit or commit any act that reflects adversely on their professional reputation, integrity, or competence.

II. Integrity of Capital Markets**A. Material Nonpublic Information**

Members and Candidates who possess material nonpublic information that could affect the value of an investment must not act or cause others to act on the information.

B. Market Manipulation

Members and Candidates must not engage in practices that distort prices or artificially inflate trading volume with the intent to mislead market participants.

III. Duties to Clients**A. Loyalty, prudence, and care**

Members and Candidates have a duty of loyalty to their clients and must act with reasonable care and exercise prudent judgment. Members and Candidates must act for the benefit of their clients and place their clients' interests before their employer's or their own interests.

B. Fair Dealing

Members and Candidates must deal fairly and objectively with all clients when providing

investment analysis, making investment recommendations, taking investment action, or engaging in other professional activities.

C. Suitability

1. When Members and Candidates are in an advisory relationship with a client, they must:
 - a. Make a reasonable inquiry into a client's or prospective client's investment experience, risk and return objectives, and financial constraints prior to making any investment recommendation or taking investment action and must reassess and update this information regularly.
 - b. Determine that an investment is suitable to the client's financial situation and consistent with the client's written objectives, mandates, and constraints before making an investment recommendation or taking investment action.
 - c. Judge the suitability of investments in the context of the client's total portfolio.
2. When Members and Candidates are responsible for managing a portfolio to a specific mandate, strategy, or style, they must make only investment recommendations or take only investment actions that are consistent with the stated objectives and constraints of the portfolio.

D. Performance Presentation

When communicating investment performance information, Members and Candidates must make reasonable efforts to ensure that it is fair, accurate, and complete.

E. Preservation of Confidentiality

Members and Candidates must keep information about current, former, and prospective clients confidential unless:

1. The information concerns illegal activities on the part of the client or prospective client,
2. Disclosure is required by law, or
3. The client or prospective client permits disclosure of the information.

IV. Duties to Employers

A. Loyalty

In matters related to their employment, Members and Candidates must act for the benefit of their employer and not deprive their employer of the advantage of their skills and abilities, divulge confidential information, or otherwise cause harm to their employer.

B. Additional Compensation Arrangements

Members and Candidates must not accept gifts, benefits, compensation, or consideration that competes with or might reasonably be expected to create a conflict of interest with their employer's interest unless they obtain written consent from all parties involved.

C. Responsibilities of Supervisors

Members and Candidates must make reasonable efforts to ensure that anyone subject to their supervision or authority complies with applicable laws, rules, regulations, and the Code and Standards.

V. Investment Analysis, Recommendations, and Actions**A. Diligence and Reasonable Basis**

Members and Candidates must:

1. Exercise diligence, independence, and thoroughness in analyzing investments, making investment recommendations, and taking investment actions.
2. Have a reasonable and adequate basis, supported by appropriate research and investigation, for any investment analysis, recommendation, or action.

B. Communication with Clients and Prospective Clients

Members and Candidates must:

1. Disclose to clients and prospective clients the basic format and general principles of the investment processes they use to analyze investments, select securities, and construct portfolios and must promptly disclose any changes that might materially affect those processes.
2. Disclose to clients and prospective clients significant limitations and risks associated with the investment process.
3. Use reasonable judgment in identifying which factors are important to their investment analyses, recommendations, or actions and include those factors in communications with clients and prospective clients.
4. Distinguish between fact and opinion in the presentation of investment analysis and recommendations.

C. Record Retention

Members and Candidates must develop and maintain appropriate records to support their investment analyses, recommendations, actions, and other investment-related communications with clients and prospective clients.

VI. Conflicts of Interest**A. Disclosure of Conflicts**

Members and Candidates must make full and fair disclosure of all matters that could reasonably be expected to impair their independence and objectivity or interfere with respective duties to their clients, prospective clients, and employer. Members and Candidates must ensure that such disclosures are prominent, are delivered in plain language, and communicate the relevant information effectively.

B. Priority of Transactions

Investment transactions for clients and employers must have priority over investment transactions in which a Member or Candidate is the beneficial owner.

C. Referral Fees

Members and Candidates must disclose to their employer, clients, and prospective clients, as appropriate, any compensation, consideration, or benefit received from or paid to others for the recommendation of products or services

VII. Responsibilities as a CFA Institute Member or CFA Candidate

A. Conduct as participants in CFA Institute Programs

Members and Candidates must not engage in any conduct that compromises the reputation or integrity of CFA Institute or the CFA designation or the integrity, validity, or security of CFA Institute programs.

B. Reference to CFA Institute, the CFA designation, and the CFA program

When referring to CFA Institute, CFA Institute membership, the CFA designation, or candidacy in the CFA Program, Members and Candidates must not misrepresent or exaggerate the meaning or implications of membership in CFA Institute, holding the CFA designation, or candidacy in the CFA Program.

Summary

LO.a: Describe the structure of the CFA Institute Professional Conduct Program and the process for the enforcement of the Code and Standards.

The Professional Conduct Program (PCP), in conjunction with the Disciplinary Review Committee (DRC), is responsible for enforcement of the Code and Standards. The CFA Institute Bylaws and Rules of Procedure for Professional Conduct form the basic structure for enforcing the Code and Standards. Professional Conduct inquiries can be prompted by: self-disclosure, written complaints and evidence of misconduct, and report by a CFA exam proctor. If the professional conduct staff believes a violation of the Code and Standards has occurred, sanctions are proposed. If the member/candidate does not accept the charges or the sanctions, the matter is referred to the DRC, which reviews materials and presentations from professional conduct staff and from the member or candidate. The DRC makes a final decision on whether there was a violation and if so what sanctions must be imposed.

LO.b: State the six components of the Code of Ethics and the seven Standards of Professional Conduct.

Members of the CFA Institute (including CFA charter holders) and candidates for the CFA designation ("Members and candidates") must:

1. Act with integrity, competence, diligence, and respect and in an ethical manner with the public, clients, prospective clients, employers, employees, colleagues in the investment profession, and other participants in the global capital markets.
2. Place the integrity of the investment profession and interests of clients above their own personal interests.
3. Use reasonable care and exercise independent professional judgment when conducting investment analysis, making investment recommendations, taking investment actions, and engaging in other professional activities.
4. Practice and encourage others to practice in a professional and ethical manner that will reflect credit on themselves and the profession.
5. Promote the integrity and viability of the global capital markets for the ultimate benefit of society.
6. Maintain and improve their professional competence and strive to maintain and improve the competence of other investment professionals.

LO.c: Explain the ethical responsibilities required by the Code and Standards, including the sub-sections of each Standard.

Covered in the next reading.

R03 Guidance for Standards I-VII

Introduction

This is a long reading, which provides guidance and recommended compliance procedures for each standard. Several examples are also given on how the standards should be applied. IFT Notes focus on the guidance and recommended compliance procedures. Read the examples (applications of the standards) from the curriculum.

Standard I: Professionalism

Standard 1 (A) Knowledge of the Law

Members and Candidates must understand and comply with all applicable laws, rules, and regulations of any government, regulatory organization, licensing agency, or professional association governing their professional activities. In the event of conflict, Members and Candidates must comply with the more strict law, rule, or regulation. Members and Candidates must not knowingly participate or assist in and must dissociate from any violation of such laws, rules, or regulations.

Interpretation:

You, as a member or candidate, must be aware of all laws where you conduct business. Stating that you are not aware of the laws and hence a violation occurred, will not be acceptable.

Guidance:

- Relationship between the Code and Standards and Applicable Law: Assume you are an investment adviser based in Malaysia. You are a Malaysian citizen and your clients are also based in Malaysia. Here the Malaysian law is the 'applicable law'. As a Level I candidate, the Code and Standards must also be considered. Let's assume that Malaysian laws prohibit participation of investment advisers in IPOs but the Code and Standards allow participation under specified circumstances, then **you have to follow the stricter law** – the Malaysian law in this case. If there is no applicable law or regulation, then Members and Candidates must follow the Code and Standards.
- Investment products and applicable laws: Follow the more strict law.
- Participation in or association with violations by others: You are responsible for violations in which you *knowingly* participate or assist. *Knowingly* is the key word here. Assume you are part of a group and you have reasonable grounds to believe a violation is taking place. Under such circumstances:
 - First, make an attempt to stop the behavior by bringing it to the notice of your supervisor/compliance department.
 - Seek the advice of independent legal counsel if the compliance department was not helpful.
 - Dissociate yourself with that activity. Dissociation varies based on your role in the

- organization; it could be:
 - Removing your name from the investment reports and recommendations.
 - Asking for a different assignment.
 - Refusing to accept a new client or continuing to advise the current client.
 - In extreme cases, leave the organization.
- Not taking an action after reporting a violation (and continuing association with the illegal activity), can be considered as participating in the illegal or unethical conduct.
- If you are not sure that a violation is taking place, then the appropriate action would be to seek the advice of legal/compliance counsel.
- CFA Institute does not compel you to report violations to the government or regulatory organization unless required by law.

Recommended procedures for compliance:

- Stay informed: Have a procedure or regular training to keep employees informed of the changes in applicable laws, rules, regulations etc.
- Review procedures: Periodically review firm's written compliance procedures to ensure it conforms to the applicable law.
- Maintain current files: Latest copies of applicable rules and regulations should be available for reference.
- Legal counsel: If in doubt how to respond to a possible violation, seek the advice of legal/compliance personnel.
- Dissociation: Document the violation if you are dissociating from an illegal activity; urge the firm to take steps to cease the activity, and resign in extreme cases.
- Advise/encourage your firm to:
 - Develop and/or adopt a code of ethics.
 - Provide information on applicable laws: make all the information regarding laws and rules available in a central location.
 - Establish procedures for reporting violations: make it easy to report violations.

Standard 1 (B) Independence and Objectivity

Members and Candidates must use reasonable care and judgment to achieve and maintain independence and objectivity in their professional activities. Members and Candidates must not offer, solicit, or accept any gift, benefit, compensation or consideration that reasonably could be expected to compromise their own or another's independence and objectivity.

Interpretation:

Maintain independence and objectivity. Do not compromise your independence and objectivity under any circumstance as it can hurt not just your firm, but the whole industry. For instance, assume you are writing a research report and the firm you are covering gives you an expensive gift. Accepting the gift may shroud your judgment to be impartial and give

an objective report.

Guidance:

- Buy-side clients: Assume you work in the research department of a large brokerage (buy-side) firm and you cover pharmaceutical firms. Your research reports are disseminated to institutional clients (buy-side clients) such as mutual funds. Mutual funds with large positions in pharma stocks might try to influence you to write positive reports. However, it is important for you, not to succumb to pressure. Independence and objectivity must be maintained.
- Investment banking relationships: Now assume your firm also has an investment banking (IB) division. Pfizer is a client of the firm, and the IB division is working closely on Pfizer's secondary offering. The IB division may influence research analysts to issue favorable research reports. But, as an analyst, you must maintain your objectivity.
- Public companies: Public companies may try to influence analysts to write positive research reports.
- Issuer-paid research: Assume a company is not being widely followed. If this company approaches you to write a research report for them, and compensates you, then there is a potential conflict of interest. The best practice for independent analysts is to negotiate a flat fee for the report. The fee should be independent of the eventual recommendation. Disclosure of the type of compensation is also important.
- Travel funding: It is best for candidates to use commercial transportation paid for by their firm, and not the client. If commercial transportation is unavailable, members and candidates may accept modestly arranged travel, to participate in appropriate information-gathering events, such as a property tour.
- Credit rating agency opinions: Credit rating agencies provide ratings for fixed-income products. If you are working at a rating agency, you may be offered incentives and compensation by the sponsoring company (companies issuing bonds) to issue a favorable rating. However, you should be objective about the analysis and ensure the processes at your agency do not result in a conflict of interest.
- Influence during the manager selection/procurement process: Assume a large pension fund is in the process of selecting an asset management company (AMC) to manage their assets. In order to get this business, AMCs may try to influence the hiring manager at the pension fund by giving gifts, etc. Irrespective of which side you are in the process (pension fund or AMC which is seeking business), do not solicit gifts or contributions either directly or indirectly that may affect your independence.
- Brokerage houses: Members and candidates hire secondary fund managers to manage specific assets, for trading and reporting. There may be attempts to influence them with gifts or compensation. It is important for members to not accept such gifts and stay objective about the hiring decision.

Recommended Procedures for Compliance:

- Protect the integrity of opinions.

- Create a restricted list for companies where a firm wants to disseminate only factual information, and no negative or positive opinion.
- Restrict special cost arrangements: use corporate aircraft only if commercial transportation is not available.
- Limit gifts: a strict limit for token gifts that can be accepted must be established.
- Restrict investments: Enforce prior approval for employees purchasing equity or equity-related IPOs.
- Review procedures.
- Establish an independence policy.
- Appointed officer: appoint a senior officer to ensure compliance with the firm's code of ethics.

Standard 1 (C) Misrepresentation

Members and Candidates must not knowingly make any misrepresentations relating to investment analysis, recommendations, actions, or other professional activities.

A misrepresentation is any untrue statement, or omission of fact, or any statement that is otherwise false or misleading.

Interpretation:

The foundation of any client-customer relationship is trust. If trust is lost because of misrepresenting facts, it not only hurts you, but confidence in the entire profession and integrity of capital markets as well.

Guidance:

- Impact on investment practice: Assume your firm has been managing a large cap equity fund for several years and has added a small cap fund last year. If the firm claims it has years of experience managing small cap funds/stocks, then it would be misrepresenting facts.
 - Members and candidates must not misrepresent facts including their qualifications, or credentials. For instance, if you have cleared only two levels of the CFA program, you cannot claim to be a CFA charterholder.
 - When issuing a research report, you may be using third-party information. You must exercise care and diligence when using third-party information such as credit ratings, research, or marketing materials, to ensure there is no misrepresentation.
 - If you are using external managers to manage specific areas, you must not represent their investment practices as your own.
- Performance reporting:
 - If you have chosen a benchmark for your portfolio, are the strategies of both comparable? Are you choosing a benchmark because it makes the portfolio's performance look better?
 - You must ensure the performance evaluation of your portfolio has a reasonable

basis.

- Provide pricing information of securities to clients on a consistent basis. Do not change pricing providers solely on the basis of higher value of a security. This is especially true of illiquid securities. This will be misrepresenting information as investors make the decision of whether or not to hold an illiquid security based on the information provided.
- **Social media**: The language used on social media platforms such as Facebook and Twitter is often informal. However, members and candidates must ensure the information provided is the same as in traditional modes of communication. The format must adhere to the Code and Standards, even though there is a great deal of anonymity.
- **Omissions**: Facts or outcomes must not be omitted, especially when it comes to performance measurement and attribution. For example, assume a manager had exceptional performance in the past three years, but negative returns in the three years preceding it. He must present the performance for the entire period and not omit years of bad performance; that is called cherry picking (or selective presentation).
- **Plagiarism**: Plagiarism is using the work of others without acknowledging or attributing the source of information. Examples include:
 - Using the research report of another firm, and then redistributing it by changing the names.
 - A research report based on multiple sources of information without naming the sources.
 - Excerpts from articles with little or no change in wording.
 - Not naming specific references, but instead attributing to “leading investment analysts”.
 - Using charts and graphs without naming their sources.
- Members and Candidates must disclose the source of information used in their reports. If it is paid for, then it must be disclosed. Sentences reproduced must be within quotes and the author named specifically.
- **Work completed for employer**: Work (models/reports/research) done within a firm may be used by others in the firm without attribution. If the person who developed a model has left the firm, the firm can continue using it as it is a property of the firm without naming the person. However, no one can claim that the work done by the person who has quit the firm has been done by the one who is now using it.

Recommended Procedures for Compliance:

- **Factual presentations**: Each member and candidate must be aware of the firm’s and the individual’s capabilities and limitations. A written list of the firm’s available services should guide the employees who present to clients.
- **Qualification summary**: Each member and candidate should prepare a summary of his/her qualifications and experience to present to clients. These must be periodically reviewed.

- Verify outside information: Ensure material from a third-party is accurate before presenting it to clients.
- Maintain web pages: Any information published on a web page must be current, and accurate.
- Plagiarism policy: Maintain copies of research reports/articles used in making your research report, attribute quotations to their source, and attribute summaries to their sources.

Standard 1 (D) Misconduct

Members and Candidates must not engage in any professional conduct involving dishonesty, fraud, or deceit or commit any act that reflects adversely on their professional reputation, integrity, or competence.

Guidance:

Any act that involves lying, cheating, stealing, or other dishonest conduct is a violation of this standard if the offense reflects adversely on a member's or candidate's professional activities. Although CFA Institute discourages any sort of unethical behavior by members and candidates, the Code and Standards are primarily aimed at conduct and actions related to a member's or candidate's professional life. Some important points based on examples seen often:

- Using alcohol during business hours, though not illegal impairs a person's ability to think objectively.
- If a member or candidate declares personal bankruptcy, it is not misconduct. But, if the circumstances that led to bankruptcy include deceit or fraud, then it would be a violation and deemed as misconduct.

Recommended Procedures for Compliance:

- Code of ethics: Adopt a code of ethics that every member must adhere to.
- List of violations: Communicate to all employees a list of potential violations and the associated sanctions.
- Employee references: Do background (reference) checks of employees to ensure they have not had a brush with the law in the past and are eligible to work in the investment profession.

Standard II: Integrity of Capital Markets

Standard II (A) Material Nonpublic Information

Members and Candidates who possess material nonpublic information that could affect the value of an investment must not act or cause others to act on the information.

Guidance:

- What is material information?: This is information that, if disclosed, can have an impact on the price of a security, or information that investors would want to know before making an investment decision. For example, information that the CEO of a company was involved in a scandal to manipulate financial statements and is going to be arrested, is material information. Other common examples include mergers and acquisitions, new product licenses, changes in management, bankruptcies, legal disputes, etc.
- What constitutes “nonpublic” information?: As the name implies, information that has not been made public is called nonpublic information. For instance, if a pharmaceutical company has just received news that a particular drug has been approved by FDA and it is not made public yet, then it constitutes nonpublic information. This is also material information as it is something investors would like to know before investing in the company.
- Mosaic theory: As per the Mosaic theory, analysts are free to act on public and nonmaterial, nonpublic information without risking violation. Let’s take an example from the curriculum. An analyst is researching a company in the furniture industry. He analyzes the public disclosures, and speaks with many furniture retailers on which he bases his recommendation report. The information gathered from furniture retailers is an example of nonmaterial nonpublic information because the information is not public, and not material by itself to influence the stock prices in any way.
- Social media: Members and candidates must ensure that information obtained from closed groups on social media (Facebook, LinkedIn) is accessible to the public through other sources.
- Using industry experts: Using experts is appropriate as long as members are not requesting or acting on material nonpublic information.
- Investment research reports: Assume you are a well-known analyst and your recommendation reports might impact stock prices. Since you are not an insider and did not base your report on insider information, Standard II (A) does not apply. In this case, you are not required to make the report public. If the public wants access to the report, they can be asked to pay for your services.

Recommended Procedures for Compliance:

- Achieve public dissemination: Take steps to publicly disseminate material nonpublic information. Ensure no investment action is taken based on the information.
- Adopt compliance procedures: Adopt compliance procedures to prevent the misuse of material nonpublic information. Ex: review employee trading, investment recommendations, and interdepartmental recommendations.
- Adopt disclosure procedures: Same information should be communicated to the market in an equitable manner. The information received by buy-side clients should be the same as sell-side clients, and the same goes for large firms and small firms.
- Issue press releases: Press releases must be made before conference calls and analyst meetings so that new information is disclosed at such gatherings.

- Firewall elements: A firewall is an information barrier created to prevent the flow of material nonpublic information within a firm; for instance, between the brokerage and investment banking departments of a firm. Listed below are a few ways a firewall is implemented:
 - Review of employee trading.
 - Route interdepartmental communications through the compliance or legal department.
 - Document how to enforce procedures to limit information flow within the firm.
 - Review/restrict proprietary trading when a firm is in possession of material nonpublic information.
- Appropriate interdepartmental communications: Document procedures for how interdepartmental communications must occur, review trading activity, and what actions to take if violations occur.
- Physical separation of departments: To prevent sensitive information flowing from one department to another. Ex: IB/corporate finance to be physically separated from sales and research of a brokerage firm.
- Prevention of personnel overlap: An employee should be on only one side of the firewall. For instance, an employee working in the commercial lending department of a bank must not be associated with its trust/research departments.
- A reporting system: Have a reporting system in which authorized people can review and approve communications between departments. If sharing of certain information is necessary across the firewall, then a designated officer must ascertain whether sharing is essential and must monitor the process.
- Personal trading limitations: Enforce restrictions on personal trading by employees. Monitor both proprietary and personal trading.
- Record maintenance: Maintain records of interdepartmental communication.
- Proprietary trading procedures: Outline procedures for under what situations, there should be restrictions on proprietary trading:
 - Market making: Restrictions on trading if the firm is a market maker can be counterproductive as it may be a signal to traders that the firm is in possession of some material nonpublic information. The firm must take the contra side of unsolicited customer trades.
 - Arbitrage trading: Must not engage in proprietary trading if it is in possession of sensitive information.
- Communicate to all employees: Educate employees through trainings on how to identify material nonpublic information and how to act (consult a supervisor/compliance officer) if they possess such information. Circulate written compliance policies and procedures to all employees.

Standard II (B) Market Manipulation

Members and Candidates must not engage in practices that distort prices or artificially inflate trading volume with the intent to mislead market participants.

What it includes:

- Disseminating false information into the market.
- Misleading market participants by distorting prices.

Guidance:

- Information-based manipulation: Spreading false rumors to induce trading by others. For example, an analyst may pump false information into the market through blogs or some other media to artificially inflate stock prices.
- Transaction-based manipulation: Transactions that artificially affect the prices or volume of a security. For example, if transactions show a security to be more liquid, then market participants perceive it favorably and may buy. For example, a large firm may have offices in Tokyo and Chicago. One office may sell a large number of shares and the other office may buy. While it may appear as if the liquidity/trading volume of the security is up. But, in reality, the trading was within the firm.

Standard III: Duties to Clients

Standard III (A) Loyalty, Prudence, and Care

Members and Candidates have a duty of loyalty to their clients and must act with reasonable care and exercise prudent judgment. Members and Candidates must act for the benefit of their clients and place their clients' interests before their employer's or their own interests.

Interpretation:

Client interests come first, followed by the employer and then personal interests of the member or candidate. The only exception is that the integrity of capital markets must take precedence over the client's interests if there is a conflict. Prudence requires caution and discretion. When handling funds of a client, prudence requires that you treat them with the same skill, care, and diligence as you would treat your own funds.

Guidance:

- Understanding the application of loyalty, prudence, and care: Investment advisers have different job roles; some have fiduciary responsibilities that are imposed by law and require a higher level of trust than other business roles. Irrespective of whether or not they are in a fiduciary role, members and candidates are expected to work in the client's best interest, and be loyal, prudent, and exercise care in managing the client's portfolio.
- Identifying the actual investment client: Identify who is the actual client. It's often easy to define a client but there are instances when it may not be clear. For example, if a pension

plan hires an investment manager, then the client is not the pension plan but the beneficiaries of the plan. In this case the hiring entity is not your client. In some cases, there may not be any direct clients or beneficiaries. Ex: a fund manager managing the fund to an index. In such cases, fund managers should invest according to the stated mandate.

- Developing the client's portfolio: Care must be taken in developing portfolios, which are consistent with the clients' objectives, circumstances, constraints, and risks. Investment decisions should be based on the overall portfolio, rather than the characteristics of an individual investment.
- Soft commission (dollar) policies: Assume a client has hired you to manage his funds. You have discretion over the selection of brokers to execute transactions. Conflicts may arise if you use client brokerage (money paid by the client for trade execution) to purchase research services from the broker. This practice is called "soft dollars" or "soft commissions." If you pay a higher brokerage commission than you would normally pay, to allow for the purchase of goods or services, without a corresponding benefit to the client, you have violated the duty of loyalty to your client.
- Proxy voting policies: Assume you are an investment manager and you have purchased 1 million shares of General Electric on behalf of your client. Since you are managing your client's portfolio, you can vote on behalf of the client. You should perform a simple cost-benefit analysis to decide whether or not to vote. When you vote it should be in the best interest of the client (shareholder), not the company management. Your firm's proxy voting policies should be disclosed to clients.

Recommended Procedures for Compliance:

- Regular account information: Submit a quarterly statement to the client that includes credits, debits, securities holdings, and transactions during the period. Indicate whether the client must hold or sell assets. And if sold, where the proceeds should be invested in and when.
- Client approval: If unsure of what course of action to take with respect to a client, members and candidates must discuss with the client in writing and take approval.
- Firm policies: Encourage firms to adopt these policies:
 - Follow all applicable rules and laws.
 - Establish the investment objectives of the client: return requirements, risk profile, experiences, and constraints.
 - Consider all the information when taking actions: client's needs and circumstances, the client's portfolio and investment's individual characteristics.
 - Carry out regular reviews: If a client's circumstances have changed (sudden need for large sums of money, or an unexpected inflow of money), then they must be addressed.
 - Deal fairly with all clients with respect to investment actions.
 - Disclose conflicts of interest.

- Disclose compensation arrangements: If a manager is compensated based on the returns generated for a client, then it must be disclosed to the client.
- Maintain confidentiality.

Standard III (B) Fair Dealing

Members and Candidates must deal fairly and objectively with all clients when providing investment analysis, making investment recommendations, taking investment action, or engaging in other professional activities.

Interpretation:

The standard focuses on dealing ***fairly and objectively*** with all clients. It does not mean equally because the circumstances of every client will be different. Also, a firm may offer different levels of services. A client paying a higher fee for a personalized service cannot be treated in an equitable manner with one who is not. Moreover, it is also not possible to communicate information to all clients at the same time as the modes of communication may vary (e-mail, phone, and fax).

Guidance:

- Investment recommendation is any opinion to buy, sell, or hold a security/investment. Guidelines on how recommendations must be disseminated to clients:
 - All your clients must have a fair opportunity to act on the investment recommendation.
 - There should **not** be selective disclosure such that your large clients receive a report first and the smaller clients later. There may be practical difficulties in reaching all clients at the exact same time because of time differences and modes of communication, but an effort must be made to communicate in an equitable manner.
 - There may be instances when you may change your recommendation. Let's assume you issued a buy recommendation for a stock erroneously. You changed it later to sell and if there are clients who have acted on the buy order but are not aware of the change to *sell*, you must advise them of the change before accepting the order.
- Guidelines for members and candidates whose primary role is to take actions based on investment recommendations received either from within the firm or external sources:
 - Take care to treat all clients fairly.
 - IPO and secondary offerings: Distribute to all clients for whom the investments are appropriate. Allocation of the stock should be consistent with the policies of the firm.
 - Oversubscribed issues: Distribute on a pro rata and round-lot basis. Refrain from buying for individual and family accounts. But, if a family-member is a fee-paying client, then the family member must be treated on an equal basis as any other client.
 - Block trade: All accounts of clients in a block trade must be given the same execution price and charged the same commission fee.

- Orders must be time stamped.
- Orders are to be executed on a first-in and first-out basis.
- Disclose to the client the allocation procedures that the firm follows.
- Members and candidates must not withhold securities of IPOs trading at a premium in the secondary market, for their benefit.

Recommended procedures for compliance:

- Develop firm policies.
 - Limit the number of people who know that a recommendation is going to be disseminated.
 - Shorten the time frame between the decision to make an investment recommendation and actual dissemination.
 - Publish guidelines for pre-dissemination behavior: Firms must be encouraged to have guidelines that prohibit personnel, who know about the recommendation, from taking action or discussing.
 - Simultaneous dissemination: Once dissemination to all clients has happened, members and candidates may follow up with individual clients.
 - Maintain a list of clients and their holdings.
 - Develop and document trade allocation procedures (discussed above).
- Disclose trade allocation procedures.
- Establish systematic account review: Conduct periodic review to ensure no client is receiving preferential treatment and trades are based on the account's objectives. If the manager is selling from one account and buying it for another account, he must document the reasons for both the transactions.
- Disclose the levels of service and the associated fees to all clients.

Standard III (C) Suitability

1. When members and candidates are in an advisory relationship with a client, they must:
 - a. Make a reasonable inquiry into a client's or prospective client's investment experience, risk and return objectives, and financial constraints prior to making any investment recommendation or taking investment action, and must reassess and update this information regularly.
 - b. Determine that an investment is suitable to the client's financial situation and consistent with the client's written objectives, mandates and constraints before making an investment recommendation or taking investment action.
 - c. Judge the suitability of investments in the context of the client's total portfolio.
2. When members and candidates are responsible for managing a portfolio to a specific mandate, strategy, or style, they must make only investment recommendations or take only investment actions that are consistent with the stated objectives and constraints of the portfolio.

Interpretation:

Determine the suitability of an investment before taking action based on the clients' circumstances and other factors. It is the responsibility of members and candidates who provide investment advice to a client to determine the suitability of an investment. Sell-side analysts and other members who execute instructions are **not** responsible for suitability analysis.

Guidance:

- Developing an investment policy: Gather client information (personal data, objectives, risk, and circumstances) at the start of the relationship. Develop an IPS that outlines return requirements, risk tolerance, and all investment constraints. IPS also outlines the roles and responsibilities of the parties in the advisory relationship, when periodic reviews will be conducted and the IPS reevaluated.
- Updating an investment policy: IPS is to be updated at least annually to reflect changes in market expectations and circumstances of the client. Needs and circumstances of the clients can change at any time and the investment recommendations/decisions must take note of this. Examples of changes in an individual's circumstances: tax status, number of dependents, liquidity needs, loss of job/change in current income, etc.
- The need for diversification: Combining different investments reduces the risk of a portfolio having all assets in a single investment. An investment that is relatively risky on its own may be suitable in the context of the entire portfolio.
- Addressing unsolicited trading requests
 - Requests from clients for trades that do not align with the risk and return objectives of a client's IPS: Members and candidates must take efforts to balance the client's request while not deviating from the IPS.
 - Unsolicited requests that are not suitable investments: If your clients ask you to make a trade that is not in accordance with the IPS, then refrain from making the trade until you discuss it with the client. Educate the client about the deviation from the current IPS.
 - If the client insists on making the trade and if you think it will have a material impact on the portfolio, update the IPS. If the client refuses to have the IPS modified, then determine the future of the advisory relationship.
- Managing to an index or mandate: Invest according to the mandate. For example, assume you are a portfolio manager for a small cap fund and your mandate is to include stocks below a certain market capitalization. You would be deviating from the mandate if you buy large cap stocks even if you expect large caps to perform exceptionally well.

Recommended Procedures for Compliance:

- Investment policy statement: Both individual and institutional investors must have an IPS. The IPS should outline the following: client identification, investor objectives, investor constraints and performance measurement benchmarks.

- Regular updates: IPS is to be updated on a regular basis (at least annually) to reflect changing circumstances and capital market expectations.
- Suitability test policies: Firms must be encouraged to have test procedures to determine the suitability of investments for different types of clients.

Standard III (D) Performance Presentation

When communicating investment performance information, members and candidates must make reasonable efforts to ensure that it is fair, accurate, and complete.

Guidance:

- Provide credible performance information to clients and prospective clients. Should not state that past performance can be obtained again.
- Avoid misstating performance or misleading clients.
- If the presentation is brief, make detailed supporting information available to clients and prospects, on request.

Recommended Procedures for Compliance:

Applying the GIPS standards is recommended, but not required. Firms that claim compliance without applying GIPS standards must do the following:

- Consider the knowledge and sophistication of the audience.
- Present the performance of the weighted composite of similar portfolios rather than using a single representative account. Assume there are three portfolios with similar mandates worth 2 million, 10 million and 8 million. If they generated returns of 9%, 2%, and 2%, respectively, then take a weighted average of returns.
- Include terminated accounts as part of performance history. Also state when those accounts were terminated.
- Include disclosures that fully explain the performance results being reported.
- Maintain the data and records used to calculate the performance being presented.

Standard III (E) Preservation of Confidentiality

Members and Candidates must keep information about current, former, and prospective clients confidential unless:

1. The information concerns illegal activities on the part of the client.
2. Disclosure is required by the law.
3. The client or prospective client permits disclosure of the information.

Guidance:

- Status of client: Even if an entity is no longer a client, members and candidates must maintain the confidentiality of client records.
- Compliance with laws: Comply with applicable law. If a client is involved in illegal activities and the applicable law requires members and candidates to maintain

confidentiality, then the information must not be disclosed.

- **Electronic information and security:** Members and candidates need to be aware of possible accidental disclosures. They should take care when communicating sensitive client information. For instance, assume two clients an investment manager is dealing with, have similar names. When sending an e-mail with updated IPS, the investment manager types in the name of the intended recipient and doesn't realize that it goes to the other client instead of the intended recipient. Such mistakes can have dire consequences.
- **Professional conduct investigations by CFA Institute:** If permissible under law, members and candidates must cooperate with PCP and provide information about a client in support of an investigation. Any information given to PCP stays confidential.

Recommended Procedures for Compliance:

- The simplest, most conservative, and most effective way to comply with Standard III (E) is to avoid disclosing any information received from a client, except to authorized fellow employees who are also working for the client.
- **Communicating with clients:** Follow firm-supported communication methods and compliance procedures when communicating confidential information.

Standard IV: Duties to Employers

Standard IV (A) Loyalty

In matters related to their employment, Members and Candidates must act for the benefit of their employer and not deprive their employer of the advantage of their skills and abilities, divulge confidential information, or otherwise cause harm to their employer.

Interpretation:

- Assume you work for an investment management firm and have committed to work 45 hours a week. During this time, you'll not indulge in any activity that will deprive your employer of your skills and abilities.
- Now, assume you are about to place a large buy order for a stock for a client. You are tempted to place an order for your own account before buying for the client. This is called front running and it must be avoided, as you must place your client and employer's interests before your own interests.
- Everything else takes precedence before duty to your own self. Of course, it's not a blanket statement that requires members and candidates to always put work ahead of personal commitments and important family obligations. The standard recommends members to enter into a dialogue with employers to strike a balance between work and personal life.

Guidance:

- The employer must not have rules/written policies that conflict with responsibilities of

members and candidates. If there are any, then you must encourage your employer to change those policies.

- Independent practice: Independent practice is engaging in a business activity where you get paid, and the work is not related to the employer. Assume you are thinking of starting an independent practice to work over the weekends or after-work hours. There are certain rules that govern this:
 - You must not start a practice that conflicts with the interests of your employer.
 - Obtain consent from your employer before starting the practice. Disclose the types of services you will render, the expected duration of the services, and the compensation.
- Leaving an employer: Assume you have submitted your resignation and decided to leave your employer. There is a one month notice period. During this period:
 - You must continue to act in the best interests of your current employer.
 - You must not reveal trade secrets to your new employer.
 - You must not misuse client lists.
 - You must not solicit existing clients to shift their business to the new employer.
 - Once you have left your current employer and are being paid by the new employer, you may seek business from old clients if you have not signed a non-compete agreement with the previous employer.
- Guidelines what for what is acceptable after starting work at a new firm:
 - It is okay to use skills and experience gained at the previous employer as they are not considered confidential. Knowledge of the names of former clients is not considered confidential.
 - One must not use anything (records/work) stored in paper or electronic format from the previous firm. Ex: Excel model for the pharmaceutical industry developed at the previous employer.
- Use of social media:
 - Follow firm policies with respect to social media for interacting with clients and prospective clients. Ex: when employees are leaving an organization, it may not be appropriate to announce it on social media as firms may have rules on how and when to announce this to clients.
 - The recommended practice is to have separate accounts for personal and professional social media activities.
 - If there are no firm rules, it's best to act in the spirit of the Standard and not engage in any activity that would harm the employer.
- Whistle-blowing: Bringing insider knowledge of illegal/unethical activities in an organization to the attention of law enforcement activities is called whistle-blowing. Whistle-blowing is not acceptable if the intent is for personal gain.
- Nature of employment: Understand the nature of employment (full-time employee or a contractor). You need to be aware of the terms of the relationship: number of hours,

compensation, benefits, work location, client expectations, etc.

Recommended procedures for compliance:

- Competition policy: Relates to the independent practice, we saw in the guidelines section. You must understand the rules/procedures of your firm with respect to pursuing an independent practice.
- Termination policy: Understand the termination policies of your employer.
- Incident-reporting procedures: Be aware of incident-reporting procedures at your firm. If there is none, encourage your firm to adopt one.
- Employee classification: Understand your status within the firm: part-time, full-time, contractor. Be aware of the policies that apply to your class.

Standard IV (B) Additional Compensation Arrangements

Members and Candidates must not accept gifts, benefits, compensation, or consideration that competes with or might reasonably be expected to create a conflict of interest with their employer's interest unless they obtain written consent from all parties involved.

Interpretation:

- Assume you are a portfolio manager working for an investment management firm. Assume a client has benefited immensely from your work, and would like to gift an expensive cruise for you and your family as a token of appreciation. This is an example of additional compensation.
- Assume you use a brokerage firm to execute orders for your clients. If the brokerage wants to send a gift so that you continue to direct business to them in the future, it is an example of additional compensation.

Guidance:

- Obtain permission before accepting compensation that might create a conflict. You must first disclose to your employer and obtain written consent for any compensation that may create a conflict.
- "Written consent" includes any form of communication that can be documented.
- Not all gifts need to be reported. For example, if a brokerage firm sends you a desktop calendar, or if a client sends you a pen as a token gift (not of significant value), then it need not be reported.
- Discuss possible limitations to their abilities to provide services that may be competing with your employer's during the negotiation and hiring process.

Recommended procedures for compliance:

- Make an immediate written report to your supervisor and compliance officer specifying any compensation you propose to receive.
- The details of the report should be confirmed by the party offering the additional compensation, including performance incentives offered by clients. In our earlier

example, where the client offered an expensive vacation, when you disclose the same to your employer, it must be validated by the client. This written report should state the terms of any agreement.

Standard IV (C) Responsibilities of Supervisors

Members and Candidates must make reasonable efforts to ensure that anyone subject to their supervision or authority complies with applicable laws, rules, regulations, and the Code and Standards.

Interpretation:

- This standard applies to anyone who has supervisory responsibilities, irrespective of whether or not the employees under their supervision are CFA Institute members, CFA charterholders, or candidates.
- If the number of employees under supervision is large, then supervisors may delegate responsibilities to subordinates, but that does not absolve them of responsibility in case a violation happens. Supervisors must ensure their subordinates are aware of the rules, applicable laws, firm policies, Code and Standards, etc.
- They must have regular training programs on compliance policies for employees under their supervision.
- If the compliance procedures at a firm are inadequate, they must bring it to the attention of the firm's senior managers.
- If the compliance procedures are inadequate or non-existent, then members and candidates should decline supervisory responsibility.

Guidance:

- System for supervision
 - Understand the compliance procedures of the firm.
 - Ensure adequate compliance procedures are in place that cover all possible violations. It is not possible to cover every potential violation that may occur.
 - Once a violation is detected, a supervisor must immediately report the misconduct and initiate an assessment to determine the extent of wrongdoing. It is not sufficient to warn the employee or rely on his/her statements that it will not recur.
 - He must also ensure that the act is not repeated until the investigation is complete.
- Supervision includes detection
 - Supervisors are responsible for detecting violations.
 - Supervisors are responsible for ensuring compliance procedures are implemented and that they are followed through periodic review.
 - Assume you as a supervisor have taken adequate steps to ensure compliance procedures are in place. Despite this, a violation occurs. Since adequate steps were taken, you as a member may not be in violation of the standard under these circumstances. However, it is an indication that the existing compliance procedures

are not sufficient.

Recommended Procedures for Compliance:

- Encourage firms to adopt a code of ethics for strong ethical foundation.
- Have adequate compliance procedures to ensure the policies in the Code and Standards, and securities laws are implemented at the firm and adhered to on a daily basis.
- Distinguish between the code of ethics and compliance procedures. Keeping them separate helps serve their individual purpose.
 - Compliance procedures must be written in plain language such that any average person can understand and assimilate them easily.
 - Assign a compliance officer who has the authority to implement the firm's compliance procedures.
 - Establish the hierarchy of supervision and assign duties among supervisors.
 - Once the compliance system is in place, supervisors must periodically monitor to detect violations, and if discovered, must take the necessary actions.
- Implementation of compliance education and training.
- Establish an appropriate incentive structure.
 - Supervisors and firms must analyze the compensation structure to see if it encourages profits at the expense of ethical behavior.
 - Is “*how*” profits are generated given less importance than “*how much*” profits are made for the firm?

Standard V: Investment Analysis, Recommendations, and Actions**Standard V (A) Diligence and Reasonable Basis**

Members and Candidates must:

1. Exercise diligence, independence and thoroughness in analyzing investments, making investment recommendations, and taking investment actions.
2. Have a reasonable and adequate basis, supported by appropriate research and investigation, for any investment analysis, recommendation, or action.

Interpretation:

The level of diligence and thoroughness of research depends on the investment philosophy the member/firm is following and the role of the member in the investment decision making process.

Guidance:

- Define diligence and reasonable basis.
- When using secondary or third-party research (research conducted by someone outside the member's firm) make reasonable efforts to ensure third-party research is sound.
- Ensure the firm has a policy about periodic review of approved third-party research providers. If not, you must encourage the firm to adopt a formal review practice.

- You may rely on the judgment of others (senior managers) in your firm if you believe the due diligence done by them was adequate.
- When using quantitatively oriented research ensure the soundness of models. You are not expected to become an expert in every technical aspect of the models, but you must understand the data, parameters, assumptions and limitations of these models. Test the output of these models under various scenarios before distributing the product. Ensure that the model includes a broad range of scenarios, even high risk and potentially negative outcomes that are not commonly encountered.
- Developing quantitatively oriented techniques: If you are involved in developing new models/algorithms, then you must exercise higher diligence in reviewing new products than individuals who would use these models. Include data for both positive and negative economic cycles. Test models using adverse volatility and performance expectations. Test the model for a wide range of input expectations.
- Selecting external advisers and sub-advisers: If you are using external advisers to manage a specific mandate, then you must diligently review them just as you would an individual fund/security. Review if the published return information is accurate. Understand the adviser's compliance procedures, investment process and if he/she adheres to the stated strategy.
- Group research and decision making: Often, members and candidates are part of a group that collectively produce an investment analysis or research. The group arrives at a consensus and gives a recommendation. The names of the members are included in the report. If you do not agree to the final recommendation, but believe that consensus opinion has a reasonable and adequate basis, and is independent and objective, then you need not dissociate yourself or ask that your name be removed from the report.

Recommended Procedures for Compliance:

- Establish a policy that research reports must have a reasonable and adequate basis. Either an individual or a review committee consisting of a group of employees must be appointed to review the report before it is circulated to the outside world.
- Develop written guidance for analysts, supervisory analysts and review committee that outline due diligence procedures if a recommendation has a reasonable and adequate basis.
- Develop criteria for assessing the quality of research.
- Develop written guidance for testing of all computer-based models.
- Develop measurable criteria for assessing outside providers. This relates to the guidelines for using external/third party research, we saw this in the previous section.

Standard V (B) Communication with Clients and Prospective Clients

Members and Candidates must:

1. Disclose to clients and prospective clients the basic format and general principles of the investment processes they use to analyze investments, select securities, and construct portfolios and must promptly disclose any changes that might materially affect those processes.
2. Disclose to clients and prospective clients significant limitations and risks associated with the investment process.
3. Use reasonable judgment in identifying which factors are important to their investment analyses, recommendations, or actions and include those factors in communication with clients and prospective clients.
4. Distinguish between fact and opinion in the presentation of investment analyses and recommendations.

Interpretation:

This standard emphasizes the need for communicating clearly and frequently with clients. It is important to communicate to clients what factors they considered while making the recommendation. If there is a change in the risk characteristics of a security or asset, then this must also be communicated.

Guidance:

- **Informing clients of the investment process:** Describe your/firm's investment-decision making process to the client. This must include the pros and cons, risks and limitations of the process.
 - Just communicating the final recommendation (for instance, buy/sell a security) to the client is not sufficient. You must explain in simple language the investment process.
 - If there is any change in the process, inform the client.
 - Communicate to the client if any external advisers are being used for their expertise to manage a specific strategy.
- **Different forms of communication:** Communication is not restricted to traditional written report. It could be in-person meetings, e-mail, telephone conversation etc.
 - Care should be taken when communicating through social media.
 - If the recommendation is concise (stock-list), then you must notify clients that additional information will soon be made available.
- **Identifying risk and limitations:** Disclose to clients the risks and limitations of the investment process/product.
- **Report presentation:** In the report include those elements that were important for analysis and conclusion.
- **Distinction between facts and opinions in reports:** Assume you are recommending a sugar stock because you believe that the government will raise the export quota limit. But, this has not been done yet; so, it is just your opinion. You must distinguish between

facts and opinion by stating this is an opinion. However, for instance, if you were presenting the performance of the past three quarters, then this would be a fact.

Recommended Procedures for Compliance:

The information included/excluded in research reports varies given the diverse nature of clients and investment assets. There is no specific checklist for what must be included. But, firms must have a rigorous methodology to review research meant for dissemination to clients.

Standard V (C) Record Retention

Members and Candidates must develop and maintain appropriate records to support their investment analyses, recommendations, actions, and other investment-related communications with clients and prospective clients.

Interpretation:

Members and candidates must retain records that support their research, analysis, and conclusion. What records to maintain depends on the member involved in the decision-making process. Records can be maintained either in hard copy or electronic format.

Guidance:

- New media records: It is the member's/candidate's responsibility to maintain a record of information posted/discussed in social media even if the firm does not have a record retention policy yet. Examples include twitter/blog posts, Facebook updates, etc.
- Records created as part of any professional activity are the property of the firm; if a member decides to leave the firm, he/she cannot take the records or supporting documents without the consent of the previous employer.
- Members cannot reuse historical research reports if the supporting documentation is not available.
- Every country/jurisdiction where you operate may have certain rules for how much data to retain. For instance, a country's regulator may call for retaining data for the past five years. Similarly, firms may also have policies for retaining research/communication records. CFA Institute recommends retaining records for at least **seven** years.

Recommended Procedures for Compliance:

- The responsibility to maintain records that support investment action generally falls with the firm rather than individuals.
- You should archive research notes and other documents that support investment-related communications.
- If the firm has policies and procedures to facilitate record retention, then you must follow them. If not, you must encourage your firm to adopt policies for preserving records.

Standard VI: Conflicts of Interest

Standard VI (A) Disclosure of Conflicts

Members and Candidates must make full and fair disclosure of all matters that could reasonably be expected to impair their independence and objectivity or interfere with respective duties to their clients, prospective clients, or employer. Members and Candidates must ensure that such disclosures are prominent, are delivered in plain language, and communicate the relevant information effectively.

Interpretation:

Conflicts occur often in the investment profession. They occur between the interests of clients, interests of employers or could be your own personal interests. Disclose the conflict of interest in plain language to employer, clients or prospective clients.

Guidance:

- Disclosure of conflicts to employers. Some examples:
 - Assume you are working in an investment management firm. You manage a client's portfolio that has performed extremely well. The client is happy and wishes to compensate you for this performance. This is a conflict of interest with other clients and you must disclose this to your employer.
 - Assume you volunteer at a charity organization that is in no way related to your work, and you are not paid for it. However, you are passionate about the work you do at this charity which keeps you busy on weekends and mentally occupied during the week, and is depriving your employer of your skills. You must disclose this potential conflict of interest to your employer.
 - Assume you hold stocks in your personal account for which your firm has a *buy* recommendation and is suitable for many clients. This may create a conflict of interest.
- Firms create policies to prevent actions that may appear as a conflict of interest. Policies include restrictions on personal trading, outside board membership etc.
- Disclosure to clients: There are numerous instances where a conflict of interest exists; these should be disclosed to clients so that they understand the cost of their investments and the benefits received by the firm. A few instances are highlighted below:
 - Assume you hold stocks of General Electric (GE). You are asking your client to buy shares of GE; it may create a potential conflict of interest as if your client buys and the stock price increases, you will benefit from the movement.
 - You receive compensation (1% commission) from your employer when you recommend certain mutual funds. You must disclose this to the client as the client may believe this recommendation is keeping the client's best interests in mind. Following the disclosure, the client may decide, whether the mutual fund is suitable or not.

- Assume you issue a buy recommendation on General Electric and recommend your client to buy the stock. If your firm also has an investment banking relationship, for instance, then it must be disclosed to the client. The client can then decide if it is in his best interest, or the interest of the firm and GE.
- Cross-departmental conflicts: Assume you are a research analyst (sell-side analyst) working at a brokerage firm. Your firm has an investment banking department, and may pressurize you to write favorable reports for companies with whom they have an existing relationship or are trying to forge one. Ideally, existing companies should be on a restricted list. But, if that is not possible, then you must make a disclosure of the investment banking relationship in the recommendation report. Another example where such conflicts may arise is buy-side analyst/ banks with underwriting powers.
- Conflicts with stock ownership: Members and candidates must disclose any material ownership in a stock/investment that they are recommending to clients.
- Conflicts as a director: There are three possible conflicts of interest if you are an investment professional and serving as a director of a company:
 - Duties owed to clients and duties owed to shareholders of the company.
 - As a director, you may receive securities/options to purchase securities of the company as compensation. A conflict may arise if trading in these securities increases the value of the security.
 - As a director, you may be privy to material nonpublic information about the company. There may be a perception that the director communicates this information to his firm and investment recommendations are based on that information.

Recommended procedures for compliance:

- Disclose special compensation arrangements with the employer that might conflict with client interests, such as bonuses based on short-term performance criteria, commissions, incentive fees, performance fees, and referral fees.
- If the firm does not permit such disclosure, you should document the request and consider dissociating from the activity. For example, if you receive a 1% bonus from your firm for selling certain mutual funds and your firm does not permit disclosing this compensation, then you should consider dissociating from the activity.

Standard VI (B) Priority of Transactions

Investment transactions for clients and employers must have priority over investment transactions in which a Member or Candidate is the beneficial owner.

Interpretation:

If you are trading for your own account, then you are the beneficial owner. But, assume, the transaction happens in your children's or spouse's account. Even though those accounts are not in your name, you benefit from them and you are the beneficial owner. The account here

applies to any account with whom you have a direct relationship.

Guidance:

- Avoiding potential conflicts: Conflicts between client's interest and investment professional's interest may occur. There is nothing unethical about managers, advisers, or mutual fund employees making money from personal investments as long as they follow these three rules:
 - The client is not disadvantaged by the trade. For example, if you are executing a sell trade, then it should not affect your clients in any way.
 - The investment professional does not benefit personally from trades undertaken for clients.
 - The investment professional complies with applicable regulatory requirements.
- Personal trading secondary to trading for clients: The order of executing trades is: clients, employers, and then your personal account, or one in which you are the beneficial owner. The rationale is to prevent personal transactions from adversely affecting the interests of clients or employers.
- Impact on all accounts with beneficial ownership: Members and Candidates may undertake transactions in accounts for which they are a beneficial owner, only after their clients and employers have had an adequate opportunity to act on a recommendation. For example, assume you are working as an investment manager and your group made a *buy* recommendation on a stock. The client must get the first opportunity to act on the recommendation, then the employer, and then you.
- How should family accounts be treated?: Assume a close family member is a client. He/she should receive the same level of service as any other client. If you have a beneficial relationship in the fee-paying account, then you may be subject to preclearance or reporting requirements of the employer.

Recommended Procedures for Compliance:

- Limited participation in equity IPOs: Some IPO (initial public offerings) issues are highly sought after and the share price rises in value significantly after the issue is brought to the market. Usually, it is a hot IPO if the supply is limited and the demand is high. Purchase of the IPO by investment professionals creates a conflict of interest in two ways:
 - It may appear as if investment personnel are taking the opportunity away from clients for personal gain. For example, assume both you and your client are allotted 200 shares in an IPO. The client had applied for 400 but only got 200. Allotment to your account will make it seem as if some opportunity was taken away from the client for your gain.
 - The party that is giving the investment professional this opportunity to participate in the IPO is possibly trying to influence him/her in the future investment decisions.
- It is recommended that members and candidates should pre-clear participation in IPOs even where there is no appearance of conflict of interest, and stay away from equity IPOs.

From a firm's perspective, it may not be right to follow a blanket policy that bans employees from IPOs. Instead, it would be appropriate to have reliable and systematic procedures in place to identify any conflict of interest, and dealt with by supervisors.

- **Restrictions on private placements:** Private placements are transactions where you get shares of a company through a private offering, and not through a public offering. The conflict of interest here is similar to that of IPOs as it may seem that this participation in private placement is a favor for future business deals. Assume you have participated in a private placement. When the investment goes public, it may seem as if you have a vested interest if you recommend the investment to clients regardless of its suitability.
- **Establish blackout/restricted periods:** To prevent front running (the practice of trading for one's personal account before client accounts), firms have blackout periods during which investment personnel cannot trade for their personal accounts. This is to safeguard the interests of the clients. The policy on blackout and restricted periods vary from firm to firm depending on their size. It can range from a total ban on trading to preventing the investment manager from front running.
- Supervisors must establish reporting procedures for investment personnel. For example:
 - Disclosure of holdings in which the employee has a beneficial interest: This should be done at the beginning of employment and at least annually thereafter.
 - Providing duplicate confirmations of transactions: Investment personnel must direct their brokers to provide duplicate copies of all the securities transactions done with them. It serves two purposes: a) discourages unethical behavior because there is an independent verification b) a clear transaction history and flow of money is available, and not just the holdings.
 - Preclearance procedures: Obtaining clearance for planned trades helps reduce conflict of interest.
- Members and candidates must disclose to investors their firm's policies about personal investing/trading. It should be in simple language that investors can understand.

Standard VI (C) Referral Fees

Members and Candidates must disclose to their employer, clients, and prospective clients, as appropriate, any compensation, consideration, or benefit received from or paid to others for the recommendation of products or services.

Interpretation:

- Assume you provide equity investment advisory service to a client. The client is now interested in making fixed income investments as well. So, you refer someone providing this service to the client. You receive a fee for this from the person/firm giving fixed income advice. As per this standard, you must disclose the fee you get for the referral.
- Another example is where you recommend your client to purchase a mutual fund, and the fund pays you a commission. You must disclose this arrangement to the client so that he/she can understand the full cost of the investment and the benefit you are receiving.

- Say you receive a reference from someone/firm, and you pay a referral fee to the party introducing the client. You must disclose to the client the fee paid for this referral.

Guidance:

- Advise the client or prospective client about any referral fees before entering into any formal agreement.
- Disclose the nature of the consideration or benefit. For example, flat fee or percentage basis, one-time fee, continuing benefit based on performance must be disclosed.

Recommended Procedures for Compliance:

- Encourage your employer to develop procedures related to referral fees. The firm may completely restrict such fees.
- Provide clients notification of approved referral fee programs and provide the employer regular (at least quarterly) updates on the amount and nature of compensation received.

Standard VII: Responsibilities as a CFA Institute Member or CFA Candidate

Standard VII (A) Conduct as Participants in CFA Institute Programs

Members and Candidates must not engage in any conduct that compromises the reputation or integrity of CFA Institute or the CFA designation or the integrity, validity, or security of CFA Institute programs.

Interpretation:

- This standard covers the conduct of CFA Institute and Members involved with the CFA Program.
- CFA Institute programs include the CFA program, certificate in investment performance measurement (CIPM), and the Claritas Investment Certificate.
- The standard covers many aspects such as cheating on any CFA Institute examinations, violating the testing policies, disclosing confidential exam information to the public and improperly using any association with the CFA Institute to further personal or professional goals.

Guidance:

- Confidential program information: Candidates may discuss non-confidential information and curriculum material with others while preparing for the exam. Examples of information that cannot be disclosed include:
 - Specific details of questions appearing in the exam.
 - Discussing what areas or formulas were tested on the exam.
- Additional CFA program restrictions:
 - There are additional policies that define allowed and disallowed actions during the exam. Testing policies include calculator policy (only two calculators are allowed) and the personal belongings policy.

- Members may participate as volunteers in various aspects of the CFA program such as grading, administering or developing the exam. But they are not allowed to:
 - Disclose any material appearing on the exam.
 - How questions are scored.
 - Any information on the exam process.
- Expressing an opinion: Members are free to express their opinion or discontent with CFA Institute regarding its policies and procedures. For example, if you say the exam was not a good representation of the curriculum, then it is not a violation of the standard. However, if you discuss specific topics or questions, then it is a violation.

Standard VII (B) Reference to CFA Institute, CFA Designation, and CFA Program

When referring to CFA Institute, CFA Institute membership, the CFA designation, or candidacy in the CFA program, Members and Candidates must not misrepresent or exaggerate the meaning of or implications of membership in CFA Institute, holding the CFA designation, or candidacy in the CFA program.

Interpretation:

- It is not intended to prohibit factual statements related to the benefits of earning the CFA designation.
- However, the merits of CFA Institute, the CFA Program, and the Code and Standards must be expressed as the opinion of the speaker.
- This standard applies to all forms of communication.
- It is not allowed to state that someone with a CFA designation will exhibit superior performance.

Guidance:

- CFA Institute membership: CFA Institute member refers to regular/affiliate members of the CFA Institute who have fulfilled the following membership requirements:
 - Remit annually to CFA Institute a completed Professional Conduct Statement.
 - Annually pay CFA Institute membership dues.
- Using the CFA designation: Once you have earned the right to use the CFA designation, you must follow the rules associated with the usage of the designation. CFA charterholders are individuals who have earned this right by completing the CFA program and have the required years of work experience. They must also satisfy the membership requirements in order to use the designation.
- Referring to candidacy in the CFA program: Candidates may refer to their participation in the CFA program. A person is a candidate if the person appears for a specified examination after completing the registration process or the person has appeared for a specified examination, but the results are yet to be announced.
- Proper usage of the CFA marks: Upon obtaining the charter, charterholders may use the CFA marks. CFA mark may be used after a charterholder's name or as adjectives, never as a noun.

- Members and candidates must encourage their firms to create templates consistent with standard VII(B).

Instructor's Note: This is the most important reading in Ethics and will represent a major percentage of the Ethics questions on the Exam. To do well on this topic you should carefully study all the examples (Applications of the Standard) in the curriculum and also do as many practice questions as possible.

Summary

LO.a: Demonstrate the application of the Code of Ethics and Standards of Professional Conduct to situations involving issues of professional integrity.

Read the “Application of the Standard” segment for each standard in the curriculum.

LO.b: Distinguish between conduct that conforms to the Code and Standards and conduct that violates the Code and Standards.

LO.c: Recommend practices and procedures designed to prevent violations of the Code of Ethics and Standards of Professional Conduct.

1.A. Knowledge of the law	Understand applicable law, rules. Comply with the more strict law. Do not knowingly participate /disassociate from such activity.
1.B. Independence and objectivity	Use reasonable care and judgment. Maintain independence and objectivity. Do not offer/solicit gifts.
1.C. Misrepresentation	Do not misrepresent facts/performance reports. Avoid plagiarism Do not omit facts.
1.D. Misconduct	Aimed at professional life; if acts like lying, cheating, stealing affects professional reputation/integrity.
2.A. Material Nonpublic Information	Do not act or cause others to act on material nonpublic information. Achieve public dissemination. Not a violation: Mosaic Theory = material public information + nonmaterial nonpublic information.
2.B. Market Manipulation	Information-based manipulation: Blogs, other media to inflate stock prices. Transaction-based manipulation: Make a security to appear more liquid.
3.A. Loyalty, Prudence and Care	Use reasonable care and exercise prudent judgment. Place client's interests before your employer or your interests. Soft dollars should benefit the client. Strive for best execution.
3.B. Fair Dealing	Deal fairly and objectively with all clients. Note: it does not state <i>equally</i> because of different levels of service. Fee-paying family member should be treated no different than any other client. Disseminate reports without being partial. Same time is not possible because of different modes of communication.

3.C. Suitability	Develop IPS. Understand client's risk profile. Update IPS periodically.
3.D. Performance Presentation	Do not misstate performance. Ensure performance information is fair, accurate and complete.
3.E. Preservation of Confidentiality	Maintain confidentiality of current, former and prospective clients unless: a) law mandates disclosure b) illegal activities by client c) client permits disclosure.
4.A. Loyalty	Avoid front running. Get written consent from employer before starting an independent practice. You cannot take proprietary information, client lists of the previous employer.
4.B. Additional Compensation Arrangements	Do not accept gifts, benefits or compensation that will create a conflict of interest.
4.C. Responsibilities of Supervisors	Ensure anyone under your supervision complies with applicable laws, rules, regulations, and Code and Standards.
5.A. Diligence and Reasonable Basis	Exercise diligence, independence and thoroughness in analyzing investments, making recommendations. Be diligent and have a reasonable basis, even when using secondary or third-party research.
5.B. Communication with Clients and Prospective Clients	Disclose to clients the investment process, identify risks and limitations. Distinguish between fact and opinion.
5.C. Record Retention	Maintain records that support your analysis, research. Code and Standards recommend storing records for at least seven years.
6.A. Disclosure of Conflicts	Make full and fair disclosure of matters that may impair independence and objectivity. Disclosure to be made in plain language. Ex of conflicts: stock ownership, director, cross departmental (IB/research) conflicts.
6.B. Priority of Transactions	Any account from which you benefit makes you the beneficial owner. Personal trading secondary to trading for clients. Establish blackout periods to prevent front running. Limit participation in IPO.
6.C. Referral Fees	Disclose referral fee to clients, prospective clients, and employers.
7.A. Conduct as participants in CFA Institute Programs	Keep questions, exam information confidential. Comply with program restriction. You may express an opinion on the difficulty of exam, curriculum etc.

7.B. Reference to CFA Institute, the CFA designation, and the CFA program	Pay annual dues and fill professional conduct statement to claim membership. References to partial designation not allowed (CFA, Level 1). Not to be used as a noun. Only use it as an adjective. CFA mark can be used if you've cleared all three levels and fulfilled the membership requirements.
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Practice Questions

1. Nargis Dilawez, CFA, works as an independent research analyst and uses various online social media sites to make announcements, recommendations and analysis of various securities. She is a resident of Country S where there is no law against posting of comments and opinions, but since her views are read globally she is worried about regulators in certain countries who impose restrictions and requirements on online communications. According to the Standards, Dilawez should:
 - A. continue to post her comments since her resident country does not impose any regulatory restrictions.
 - B. discontinue immediately and wait for the restrictions to ease in the nonresident countries.
 - C. seek guidance from appropriate, knowledgeable, and reliable sources to diligently follow legal and regulatory trends affecting her professional responsibilities.
2. Wynona Fritz works for Brady Brokerage as a fixed income analyst. She is also registered to take the Level III examination. After analyzing both the qualitative and quantitative aspects of Saber Inc., Fritz concludes that the company is not correctly rated by the credit rating agency and should be downgraded due to the leverage in its capital structure. A senior manager from the investment banking department informs her that Saber Inc. has chosen Brady Brokerage as one of the firms to underwrite and market their new bond issue. Fritz is concerned that her report will cause the company to terminate their relationship with Brady and affect her employment. According to the Standards, Fritz should:
 - A. dissociate from the report, the underwriting, and the client.
 - B. be independent and objective in her analysis based solely on the company's fundamentals.
 - C. change her recommendation about the credit rating to remove the conflict.
3. Julie Grosky, CFA, works for Harvest Mutual Fund where she manages a fixed income fund. In a hastily compiled performance review, Grosky reports to her clients that her fund has exceeded the benchmark by 0.20%. Stuart Brennan is a client of Harvest, who writes back to inform Grosky that the fund actually underperformed the benchmark. Grosky incorrectly blames the error on a computer program newly implemented at Harvest. Grosky *least likely* violated the Standard relating to:
 - A. Misrepresentation.
 - B. Misconduct.
 - C. Independence and Objectivity.
4. Steve Miller is enrolled as a candidate in the CFA Program. He works as an assistant manager in Trust Investment Bank. He enjoys drinking liquor during his lunch break.

Miller's colleagues have noticed that he is visibly intoxicated after the lunch break and is not in a position to make rational investment decisions. Miller *most likely* violates the Standard of:

- A. Misconduct.
- B. Knowledge of the Law.
- C. Disclosure of Conflicts.

5. Isaac Dobrogost, a candidate in the CFA Program, works as an investment advisor for Zenith Mutual Fund. He is invited by one of his clients, Sahara Inc. (SI), a manufacturing company, to meet with the finance director along with a few large stakeholders of SI. In the meeting Dobrogost finds out that the company is going through a lean period and will announce a decrease in earnings in their next quarter financial results. Can Dobrogost use this information to change the rating of the company from "buy" to "sell"?
 - A. No.
 - B. Yes, because this information is given directly by the company.
 - C. Yes, because it has been disseminated to the other stakeholders as well.
6. Weinberg Inc., a global asset management company, has a large position in Wessner Pharma. The trading volume of this stock is low. In order to boost the liquidity of the stock, multiple trading desks at Weinburg start buying and selling Wessner shares from each other. The CFA Institute Standard *most likely* violated by Weinberg is:
 - A. Market Manipulation.
 - B. Misconduct.
 - C. Acting on Non Public Information.
7. Janice McDowell, CFA, is the chief investment officer of Zenith Investment Bank and wants to improve the diversification of one of its balanced funds in order to improve its returns. The investment policy statement of the fund mentions low risk investments in large-cap equities, government bonds of AA ratings and corporate bonds of high investment grade ratings. However, a new IPO offering of a small pharmaceutical company with high growth potential, promises high returns since the issue is being offered at a discount. He immediately allocates some portion of the issue to his fund, without exceeding the limit on the equity exposure of this fund. McDowell has *least likely* violated the CFA Institute Standards of Professional Conduct relating to:
 - A. Loyalty, prudence and care.
 - B. Suitability.
 - C. Fair dealing.
8. Eileen Connors is a chief trader for Ascot Investments, a money management firm. She has been told recently by her most lucrative client Shelby Company that if the performance of its accounts did not improve they will be forced to change their money

managers. Connors has purchased certain securities a few days back, whose price has gone up significantly. She has failed to allocate these trades due to her busy schedule. After the threat from Shelby, she decides to allocate the profitable trades to Shelby's account, while spreading the losing trades to other Ascot's accounts. Has Connors violated *any* Standard?

- A. Yes, related to Fair Dealing.
- B. No.
- C. Yes, related to Diligence and Reasonable Basis.

9. Jessica Morales works as an investment adviser for Chris Crosby, a middle-aged, risk averse investor. As per the investment policy statement, Morales invests in low-risk, high-income equities for Crosby keeping in mind his current needs and objectives. Recently Crosby's mother passed away leaving him with a significant inheritance. Morales continues to invest as before without any change in the investment strategy. According to the CFA Institute Standards of Professional Conduct, Morales should:

- A. stay abreast of changes in the client's net worth and accordingly update the investment policy to reflect changes in investment objectives.
- B. consider the long term aspect of Morales' investments and continue with the current strategy.
- C. keep changing the asset allocations in line with market changes.

10. Samina Haq a CFA candidate, works for Superior Trust Company. While reviewing the performance of one of the trust funds, she finds out that the trust fund has on an average performed at 5% for the last three years yet the brochure of her fund advertises an annual compound growth rate of 20%, which happened only in the past year. It also boasts of a consistent increment in the investment value above the entire market which also took place during last year. Haq's *highest* priority in avoiding a violation of the CFA Institute Standards of Professional Conduct is to:

- A. correct the performance calculation and length of time.
- B. continue with the advertisement since it did rise above the market.
- C. use the firm's average rate of return in her marketing material for all accounts.

11. Penelope Cox is employed by Jameason Investment, and provides investment advice to the trustees of SYU University in order to recommend investments that would generate capital appreciation in endowment funds. Cox has been given internal reports by the trustees that highlight the expansion of the university. Cox is approached by Bradley Cooper, a local philanthropist who is considering a generous contribution to SYU and another university in the area, but he would like to see the expansion plans of SYU before making the donation. Cox knows that he does not want to speak to the trustees hence she gives a copy of the internal report to Cooper. Has Cox violated the Code and Standards?

- A. No.

- B. Yes, preservation of confidentiality.
 - C. Yes, loyalty.
12. Lara Whitman, CFA, worked for Rapid Results Brokerage Company (RRBC) as a trader. She recently resigned her position as a trader to join another competing investment and brokerage firm. Whitman did not sign any non-compete agreement while at RRBC that would have prevented her from soliciting former clients. Whitman, however, had saved her client list and records while working at RRBC, in her personal computer at home as a second copy. She accesses this file to contact her former clients in her new job. The Standard *most likely* violated is:
- A. Loyalty.
 - B. Duties to Clients.
 - C. Communications with Clients and Prospective Clients.
13. Robert Blake is on the board of directors of Rice Industries and receives free tickets at the end of each quarter for his entire family to travel to any city of their choice in Europe for his services to the board. Blake does not disclose this information to his employer since it is not a monetary compensation. Has Blake violated any CFA Institute Standards of Professional Conduct?
- A. No.
 - B. Yes, he has to inform his employer of the benefit he receives.
 - C. Yes, because he has bought stock of Rice for some of his clients where appropriate.
14. Anna Becker is employed by Jergen Investment Management Company (JIMC). Becker is a Level II candidate and is the only CFA candidate employed by JIMC. Becker is given supervisory responsibilities of the compliance department and asked to review the firm's compliance policies and procedures, which she finds inadequate. She voices her concerns during a meeting with the CEO, who tells her to submit her recommendations in a report but these will not be implemented since the firm is undergoing a change in structure and no compliance changes will be entertained till then. According to the Code and Standards, Becker should:
- A. decline to accept supervisory responsibilities.
 - B. accept supervisory responsibilities and lay down the compliance policies and procedures for future.
 - C. wait till a new structure is implemented and then review the entire firm.
15. Greg Lou has been asked by his firm, Binkley Investment Management, to find an adviser for one of its funds which invests in derivatives and complex securities. Lou selects 12 firms based on their annual total return performance and finalizes on the adviser with the highest annual total return. Which CFA Institute Standards of Professional Conduct did Lou violate?

- A. Communications with Clients and Prospective Clients.
 - B. Professional Misconduct.
 - C. Diligence and Reasonable Basis.
16. Alex Karachanis, CFA, is an independent financial advisor with a roster of over 100 clients. Along with advisory services, he also facilitates in executing the trades for his clients and manages their portfolio. Adonia Papadakis signed up Alex in November 2013 to advise and manage her portfolio. After detailed discussions on Adonia's circumstances and return requirements, it was agreed that only large cap equity investments will be made. In mid-2013 Alex felt that large cap stocks were excessively overvalued and shifted 50% of the portfolio to small-cap stocks. Over the next six months, small-cap stocks significantly outperformed large cap stocks. It is now January 2014 and Adonia has just received her account statement for 2013. She is very happy with the performance of her portfolio. Which Standard did Alex *least likely* violate?
- A. Performance Presentation.
 - B. Communication with Clients and Prospective Clients.
 - C. Loyalty, Prudence and Care.
17. Raza Jaffery works as an independent analyst for the medical equipment industry. His reports are based on an analysis of customer interviews, manufacturers, on-site company visits, and secondary research from other analysts. Jaffery does not maintain any records or files for the information he collects but he mentions the source of his research in his reports. If the clients need information on the specific web sites, Jaffery always provides them with the relevant information. Jaffery *most likely* violated which of the following Standards?
- A. Record Retention.
 - B. Diligence and Reasonable Basis.
 - C. Misrepresentation.
18. Carla Simone, a CFA candidate and a research analyst, follows firms in the beverage industry. She has been recommending the purchase of Citrus, because of its introduction of a popular new drink for athletes and exercise enthusiasts. Simone's husband has inherited from a relative, the stock of Citrus worth \$3.5 million. Simone has been asked to write a follow up report on Citrus. She writes the report and gives a strong buy recommendation. The report does not mention her husband's ownership of the stock. Has Simone violated the CFA Institute Standards?
- A. No.
 - B. Yes, disclosure of conflicts.
 - C. Yes, independence and objectivity.
19. Izzy Zubeika, CFA, works for Topworth Mutual Fund and is a portfolio manager for an
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aggressive growth equity fund. She is planning to sell a large portion of her investment to meet the medical costs of her ailing husband. Zubeika wants to sell her stake in Royal Beverages, but her firm has recently upgraded the stock from “hold” to “buy”. Nevertheless after receiving approval from her employer she informs her broker to conduct the trade. Has Zubeika violated any CFA Institute Standards of Professional Conduct?

- A. Yes related to Market Manipulation.
- B. Yes, related to Priority of Transactions.
- C. No.

20. Dave Daisuke, CFA, works in the corporate finance department of Advile Securities. He receives a non-cash compensation for every referral he makes to the brokerage department. This arrangement is an accepted norm within the company but the clients are not informed because no cash is given out within the firm for interdepartmental referrals. According to the CFA Institute Standards, the *most appropriate* action to take for the firm to avoid a violation is to:

- A. adjust the non-cash compensation in the salaries of the personnel including Daisuke who are referring clients to the brokerage department.
- B. disclose to clients at the time of a referral, the referral arrangements within Advile’s departments.
- C. stop the referral policy to remove any conflicts of interest.

21. Lauren Crawley is enrolled to take the Level I exam. As he tries hard to remember a formula to complete a question, he notices that the person in front of him gets up to drink water and a piece of paper slips from his pocket and falls on Crawley’s table. In order to avoid a violation of the CFA Institute Standards of Professional Conduct, the *least appropriate* action taken by Crawley is to:

- A. remove it without looking at it and call the proctor.
- B. immediately call the proctor to her table and have the paper removed.
- C. look at the paper and then remove it before anyone else notices it.

22. Ankit Aacharya, CFA, while making the marketing material for his firm Aakash Capital writes in the brochure, “Aakash Capital is committed to achieving excellent performance for its clients. It hires the most eligible personnel in the field of investment management. Most of the employees have either completed the CFA Program or are enrolled as candidates in the CFA Program. As a CFA charterholder, I am the most qualified to manage client investments.” Aacharya *most likely* violated the Standard with improper references to the:

- A. CFA Designation.
- B. CFA Program.
- C. CFA Institute.

Solutions

1. C is correct. According to Standard I(A) Knowledge of the Law Dilawez should adopt the stricter law.
2. B is correct. Fritz should be independent and objective in her report. Alternatively, Brady Brokerage could place Saber Inc. on a restricted list and issue only factual information. Standard I(B) Independence and Objectivity.
3. C is correct. Standard I(B) Independence and Objectivity involves members and candidates not accepting any gifts or benefits that could be expected to compromise their independence and objectivity. Since no benefits were received Grosky has least likely violated I(B). Grosky most likely violated the Standards I(C) Misrepresentation, and I(D) Misconduct because she knowingly misrepresents the cause of the error.
4. A is correct. Refer to Standard I(D) Misconduct.
5. A is correct. If the information is not publicly disseminated by the company and Dobrogost uses it, then it becomes material nonpublic information, hence a violation of Standard II(A). A small group of stakeholders does not qualify as the public. He cannot use the information.
6. A is correct. Refer to Standard II(B) Market Manipulation. Weinberg created an appearance of greater liquidity of stock through its trading strategy and was able to manipulate the market.
7. C is correct. The Standards related to III(A) Loyalty, Prudence, and Care and III(C) Suitability are violated. The IPS mentions low risk securities, and describes the asset classes. Therefore investment in the pharma stock may not be suitable for this portfolio.
8. A is correct. Connors has violated Standard III(B) Fair Dealing by failing to deal fairly with all her clients in taking these investment actions.
9. A is correct. Refer to Standard III(C) Suitability.
10. A is correct. According to Standard III(D) Performance Presentation Haq needs to correct the calculation and length of time specifying the performance of her trust fund.
11. B is correct. Cox was given the internal reports by the trustees; because the information was confidential Cox should have refused to divulge it to Cooper. Therefore by handing the internal reports to him Cox violates Standard III(E) Preservation of Confidentiality.

12. A is correct. Standard IV(A) Loyalty is most likely violated. A member cannot take records or work performed on behalf of the firm in paper copy or electronically without permission to another firm. In this case she cannot use the firm's records of clients without the firm's permission.
13. B is correct. Blake has violated Standard IV(B) Additional Compensation Arrangements by failing to disclose to his employer benefits received in exchange for his services on the board.
14. A is correct. According to Standard IV(C) Responsibilities of Supervisors, a member or candidate should decline in writing to accept supervisory responsibilities until reasonable compliance procedures are laid down by a firm for her to assume and exercise responsibility.
15. C is correct. Lou violated Standard V(A) Diligence and Reasonable Basis by not conducting sufficient review of potential firms.
16. A is correct. Standard III(D) Performance Presentation is not violated as Alex sends a quarterly itemized statement of the funds and securities in his custody, and the transactions that occurred during this period. Standard V(B) Communication with Clients and Prospective Clients is violated because Alex should have discussed the change with the client before moving to small cap stocks. Standard III(A) Loyalty, Prudence, and Care is violated because small cap stocks might not correspond to client's risk profile.
17. A is correct. Refer to Standard V(C) Record Retention. Jaffery must carefully document and maintain copies of all information that goes in his reports in order to avoid violation of Standard V(C).
18. B is correct. Simone must disclose her husband's ownership of the stock to avoid violation of Standard VI(A) Disclosure of Conflicts.
19. C is correct. No violation has occurred because she has received approval from her employer. Standard VI(B) Priority of Transactions does not limit transactions of employees which are different from the current recommendations as long as they do not disadvantage the current clients.
20. B is correct. Disclosure to clients is important even if the referrals result in a noncash compensation. Refer to Standard VI(C) Referral Fees
21. C is correct. Refer to Standard VII(A) Conduct as Participants in CFA Institute Programs.

22. A is correct. CFA Institute and CFA Designation were improperly referenced. Refer to Standard VII(B) Reference to CFA Institute, the CFA Designation, and the CFA Program.

R04 Introduction to GIPS

Introduction

This reading focuses on:

- Why the GIPS standards were created and who it applies to.
- What are composites and what is their purpose in performance reporting.
- Who can claim compliance with GIPS, and is there a verification process.

Note: most of the material presented here is taken from the curriculum.

1. Why Were the GIPS Standards Created?

GIPS standards were created to make it easier to compare different investment management firms. Without a standard, different firms would select the method which would make their performance look better.

The GIPS standards are a practitioner-driven set of ethical principles that establish a standardized, industry-wide approach for investment firms to follow in calculating and presenting their historical investment results to prospective clients. The GIPS standards ensure fair representation and full disclosure of investment performance. In other words, the GIPS standards lead investment management firms to avoid misrepresentations of performance and to communicate all relevant information that prospective clients should know in order to evaluate past results.

Misleading practices in the absence of GIPS include:

- **Representative accounts:** Showcasing only the top-performing portfolio to represent the firm's overall investment results.
- **Survivorship bias:** Excluding poorly performing portfolios and presenting an average performance history.
- **Varying time periods:** Presenting performance for a selected time-period during which the mandate produced excellent returns.

2. Who Can Claim Compliance?

Any investment management firm can claim compliance. Complying with the GIPS standards is voluntary. Only investment management firms that actually manage assets can claim compliance. Plan sponsors and consultants cannot claim compliance unless they manage assets for which they claim compliance.

Compliance is a firm-wide process that cannot be achieved on a single product or composite. A firm has only two options with regard to compliance with the GIPS standards:

- Fully comply with all requirements of the GIPS standards and claim compliance through the use of the GIPS Compliance Statement; or
- Not comply with all requirements of the GIPS standards and not claim compliance

with, or make any reference to, the GIPS standards.

3. Who Benefits from Compliance?

Two groups benefit from GIPS compliance: investment management firms and prospective clients.

Following the GIPS standards allows **investment management firms** to assure prospective clients that the historical track record they report is both complete and fairly presented. Compliance enables the GIPS-compliant firm to participate in competitive bids against other compliant firms throughout the world. Achieving and maintaining compliance may also strengthen the firm's internal controls over performance-related processes and procedures.

Investors have a greater level of confidence in the integrity of performance presentations of a GIPS-compliant firm. Investors can easily compare performance presentations from different investment management firms.

Compliance with the GIPS standards does not eliminate the need for due-diligence, but it enhances the credibility of investment management firms that have chosen to undertake this responsibility.

4. Composites

One of the key requirements of the GIPS standards is the use of composites. A composite is an aggregation of one or more portfolios managed according to a similar investment mandate, objective, or strategy. Assume a firm manages portfolios based on one of the following strategies:

- Strategy A: Aggressive growth. Under this strategy the firm selects small cap stocks with strong growth potential over the next few years.
- Strategy B: Large cap value. Under this strategy large-cap value stocks are selected.

Give this scenario the firm should create a composite for Strategy A, and another composite for Strategy B.

A composite must include all actual, fee-paying, discretionary portfolios managed in accordance with the same investment mandate, objective, or strategy. By "actual" we mean these should be real and not dummy/model portfolios to simulate the returns. Portfolios for which the client does not pay a fee must not be included. For instance, there may be charitable organizations that do not pay a fee for their assets being managed. These should not be included. By "discretionary", we mean the investment management firm has the right to determine and purchase suitable securities for a portfolio. If there is a portfolio where the client determines what securities should be purchased, then it is non-discretionary.

The determination of which portfolios to include in the composite should be done according to pre-established criteria (i.e., on an ex-ante basis), not after the fact. This prevents a firm from including only the best performing portfolios. This is important because performance

numbers are reported based on composites.

5. Verification

Firms that claim compliance with the GIPS standards are responsible for their claim of compliance and maintaining that compliance. Once a firm claims compliance with GIPS, they may voluntarily hire an independent third-party to perform verification. Just as GIPS compliance is voluntary, verification is also voluntary. Verification is performed with respect to an entire firm. It is not done on composites, or individual departments.

Verification must be performed by an independent third party. A firm cannot perform its own verification. Third-party verification brings additional credibility to a firm's claim of compliance. To understand why a firm would pay to be verified if it is voluntary, assume there are two firms, A and B. Firm A claims GIPS compliance and firm B claims GIPS compliance with third-party verification from a reputed firm. As an investor, which firm would you be more comfortable with? Obviously, firm B, and that gives firm B an advantage over firm A.

Verification tests include:

- Whether the investment firm has complied with all the composite construction requirements of the GIPS standards on a firm-wide basis, and
- Whether the firm's policies and procedures are designed to calculate and present performance in compliance with the GIPS standards.

6. The Structure of the GIPS Standards

The provisions within the 2010 edition of the GIPS standards are divided into nine sections:

0. Fundamentals of Compliance
1. Input Data
2. Calculation Methodology
3. Composite Construction
4. Disclosure
5. Presentation and Reporting
6. Real Estate
7. Private Equity
8. Wrap Fee. Separately Managed Account (SMA) Portfolios

Summary

LO.a: Explain why the GIPS standards were created, what parties the GIPS standards apply to, and who is served by the standards.

The GIPS standards were created to avoid misrepresentation of performance. They apply to investment management firms. They are intended to serve prospective and existing clients of the investment firms.

LO.b: Explain the construction and purpose of composites in performance reporting.

A composite is an aggregation of one or more portfolios managed according to a similar investment mandate, objective, or strategy. When used in performance reporting, composites help clients evaluate how well a company has performed with different investment styles.

LO.c: Explain the requirements for verification.

Verification is performed with respect to an entire firm. It is not done on composites, or individual departments.

Verification must be performed by an independent third party. A firm cannot perform its own verification.

Practice Questions

1. Who can *most likely* claim compliance with GIPS Standards?
 - A. CFA charterholders.
 - B. Individuals.
 - C. Investment management firms.
2. According to the Global Investment Performance Standards (GIPS), the criteria for including portfolios in composites is:
 - A. all actual fee-paying, discretionary portfolios must be included in at least one composite.
 - B. all discretionary portfolios must be included in a composite.
 - C. all actual fee-paying portfolios must be included in a composite.
3. Verification of GIPS compliance is:
 - A. done by an independent third-party.
 - B. mandatory to claim compliance.
 - C. done voluntarily by the firm under the supervision of the CFA Institute.

Solutions

1. C is correct. Only investment management firms can claim compliance. Individuals, charterholders, and software to implement the Standards cannot claim compliance. Section 3. LO.a.
2. A is correct. All actual fee-paying, discretionary portfolios must be included in at least one composite. Actual non-fee paying discretionary portfolios may be included in at least one composite with appropriate disclosure. Section 5. LO.b.
3. A is correct. A firm cannot perform its own verification. Section 6. LO.c.

R05 The GIPS Standard

Introduction

In this reading we focus on:

- Goals of the GIPS Executive Committee.
- Key features of the GIPS standards and the fundamentals of compliance.
- How historical performance record must be presented.
- The nine major sections of the GIPS standard.

Note: most of the material presented here is taken from the curriculum.

Goals of the GIPS Executive Committee

The goals of the GIPS Executive Committee are presented below:

- To establish investment industry best practices for calculating and presenting investment performance that promote investor interests and instill investor confidence.
- To obtain worldwide acceptance of a single standard for the calculation and presentation of investment performance based on the principles of fair presentation and full disclosure.
- To promote the use of accurate and consistent performance data.
- To encourage fair, global competition among investment firms without creating barriers to entry; and
- To foster the notion of industry “self-regulation” on a global basis.

Key Features of the GIPS Standards

The key features of GIPS are as follows:

- The GIPS standards are the ethical standards for investment performance presentation to ensure fair representation and full disclosure of investment performance. In order to claim compliance, firms must adhere to the requirements included in the GIPS standards.
- Meeting the objectives of fair representation and full disclosure is likely to require more than simply adhering to the minimum requirements of the GIPS standards. Firms should also adhere to the recommendations to achieve best practice in the calculation and presentation of performance.
- The GIPS standards require firms to include all actual, discretionary, fee-paying portfolios in at least one composite defined by investment mandate, objective or strategy in order to prevent firms from cherry-picking their best performance. This means firms must create composites and each actual, fee-paying, discretionary portfolio must be in at least one composite. If a portfolio is not included in any composite, then the returns associated with it will not be included.
- The GIPS standards rely on the integrity of input data. The accuracy of input data is

critical to the accuracy of performance presentation. The underlying valuations of portfolio holdings drive the portfolio's performance. It is essential that these and other inputs are accurate. The GIPS standards require firms to adhere to certain calculation methodologies and to make specific disclosures along with the firm's performance.

- Firms must comply with all requirements of the GIPS standards.

Historical Performance Record

- A firm is required to initially present, at a minimum, five years of annual investment performance that are compliant with the GIPS standards. If the firm or the composite has been in existence less than five years, the firm must present performance since the firm's inception or the composite's inception date.
- Later the firm must present an additional year of performance each year, building up to a minimum of 10 years of GIPS-compliant performance.
- Firms may link non-GIPS compliant performance to their GIPS-compliant performance provided that only GIPS-compliant performance is presented for periods after 1 January, 2000 and the firm discloses the periods of non-compliance.
- Firms that manage private-equity, real estate, and/or wrap fee/separately managed account (SMA) portfolios must also comply with sections 6, 7, and 8 respectively, of the provisions of the GIPS standards that became effective as of 1 January, 2006.

Consider Firm A which was established in 2013 and decides to become GIPS compliant in 2014. Initially, it must present performance information for 2013 and 2014. Later, it must keep adding performance data every year until it builds up to a minimum of 10 years.

Consider Firm B which was established in 2008 and decides to become GIPS compliant in 2014. Initially, it must present performance information for five years from the period ending in 2010 to the period ending in 2014. Later, it must also add an additional year until it has 10 years of GIPS-compliant performance.

Compliance

The definition of the firm is the foundation for firm-wide compliance and creates defined boundaries whereby total firm assets can be determined.

- Firms must take all steps necessary to ensure that they have satisfied all the requirements of the GIPS standards before claiming compliance.
- Firms are strongly encouraged to perform periodic internal compliance checks.
- Firms may choose to have an independent third-party verification that tests the construction of the firm's composites as well as the firm's policies and procedures as they relate to compliance with the GIPS standards. Verification is recommended, but it is voluntary and must be done by a third-party verifier.
- In addition to verification, firms may also choose to have specifically focused composite testing (performance examination) performed by an independent third-

party verifier to provide additional assurance regarding a particular composite.

- The effective date for the 2010 edition of the GIPS standards is 1 January, 2011. Compliant presentations that include a performance for periods that begin on or after 1 January, 2011 must be prepared in accordance with the 2010 edition of the GIPS standards.

Implementing a Global Standard

- The presence of a local sponsoring organization for investment performance standards is essential for effective implementation and ongoing support of the GIPS standards within a country. The curriculum lists GIPS country sponsors for a few countries.
- Country sponsor ensures that the country's interests are taken into account as the GIPS standards are developed.
- The GIPS executive committee strongly encourages countries without an investment performance standard to promote the GIPS standards as the local standard. If there is no local standard in a country, then it is ideal for the country to promote and adopt GIPS as it is a global standard, and when required translate them into the local language.
- Compliance with applicable law and rules does not necessarily mean compliance with GIPS.
- **In cases in which laws and/or regulations conflict with the GIPS standards, firms are required to comply with the laws and regulations and make full disclosure of the conflict in the compliant presentation.** Firms can still comply with GIPS but they must disclose wherever there is a deviation that the law of the land is being followed.

Nine Major Sections of the GIPS Standards

The provisions within the 2010 edition of the GIPS standards are divided into nine sections:

0. Fundamentals of Compliance
1. Input Data
2. Calculation Methodology
3. Composite Construction
4. Disclosure
5. Presentation and Reporting
6. Real Estate
7. Private Equity
8. Wrap Fee/Separately Managed Account (SMA) portfolios

The provisions are categorized into requirements and recommendations. All the requirements must be implemented to claim compliance.

0. Fundamentals of Compliance

Several core principles create the foundation for the GIPS standards, including properly defining the firm, providing compliant presentations to all prospective clients, adhering to applicable laws and regulations, and ensuring that information presented is not false or misleading. Two important issues that a firm must consider when becoming compliant with the GIPS standards are the definition of the firm and the firm's definition of discretion. The definition of the firm is the foundation for firm-wide compliance and creates defined boundaries whereby total firm assets can be determined. The firm's definition of discretion establishes criteria to judge which portfolios must be included in a composite and is based on the firm's ability to implement its investment strategy.

Sample firm definition: XYX Investment Firm is a balanced portfolio investment manager that invests solely in US-based securities. XYZ Investment Firm is defined as an independent investment management firm that is not affiliated with any parent organization.

The requirements of Provision 0 are presented below (reproduced from the curriculum).

0.A.1: Firms must comply with all the requirements of the GIPS standards, including any updates, Guidance Statements, interpretations, Questions & Answers (Q&As), and clarifications published by CFA Institute and the GIPS Executive Committee, which are available on the GIPS standards website (www.gipsstandards.org) as well as in the GIPS Handbook.

0.A.2: Firms must comply with all applicable laws and regulations regarding the calculation and presentation of performance.

If there are no applicable laws and rules, then GIPS compliant firms must follow the GIPS standards. But, if a country has applicable laws and regulations regarding the calculation and presentation of performance, then the country laws must be followed and any differences with GIPS must be documented.

0.A.3: Firms must not present performance or performance-related information that is false or misleading.

0.A.4: The GIPS standards must be applied on a firm-wide basis.

0.A.5: Firms must document their policies and procedures used in establishing and maintaining compliance with the GIPS standards, including ensuring the existence and ownership of client assets, and must apply them consistently.

0.A.6: If the firm does not meet all the requirements of the GIPS standards, the firm must not represent or state that it is "in compliance with the Global Investment Performance Standards except for . . ." or make any other statements that may indicate partial compliance with the GIPS standards.

Firms are not allowed to claim partial compliance with the GIPS standards.

0.A.7: Statements referring to the calculation methodology as being “in accordance,” “in compliance,” or “consistent” with the Global Investment Performance Standards, or similar statements, are prohibited.

0.A.8: Statements referring to the performance of a single, existing client portfolio as being “calculated in accordance with the Global Investment Performance Standards” are prohibited, except when a GIPS-compliant firm reports the performance of an individual client’s portfolio to that client.

0.A.9: Firms must make every reasonable effort to provide a compliant presentation to all prospective clients. Firms must not choose to whom they present a compliant presentation. As long as a prospective client has received a compliant presentation within the previous 12 months, the firm has met this requirement.

0.A.10: Firms must provide a complete list of composite descriptions to any prospective client that makes such a request. Firms must include terminated composites on the firm’s list of composite descriptions for at least five years after the composite termination date.

A composite must be clearly described as to what mandate or strategy it is following: For example, small-cap equity value, Japan-equity, fixed-income etc. If a composite was terminated, then its performance must be presented for at least five years after termination to overcome survivorship bias.

0.A.11: Firms must provide a compliant presentation for any composite listed on the firm’s list of composite descriptions to any prospective client that makes such a request.

A compliant presentation is one that complies with all the GIPS provisions.

0.A.12: Firms must be defined as an investment firm, subsidiary, or division held out to clients or prospective clients as a distinct business entity.

Scenario: Assume there is a firm called UBL and it has a subsidiary called UBL-Asset Management (UBL-AM). If investors approach UBL-AM to trade and invest in securities/funds, then UBL-AM is the firm here, and must comply with the GIPS standards.

0.A.13: For periods beginning on or after 1 January 2011, total firm assets must be the aggregate fair value of all discretionary and non-discretionary assets managed by the firm. This includes both fee-paying and non-fee-paying portfolios.

Do not confuse this with a composite. A composite must include only actual, fee-paying discretionary portfolios. But, when a firm reports its total assets, it must include the fair value of all discretionary and non-discretionary assets and all fee-paying and non-fee paying portfolios.

0.A.14: Total firm assets must include assets assigned to a sub-advisor provided the firm has discretion over the selection of the sub-advisor.

0.A.15: Changes in a firm’s organization must not lead to alteration of historical composite

performance.

0.A.16: When the firm jointly markets with other firms, the firm claiming compliance with the GIPS standards must be sure that it is clearly defined and separate relative to other firms being marketed, and that it is clear which firm is claiming compliance.

Outlined above are requirements. The GIPS standards also include requirements. For example, assume an investment management company, HS, has different geographical offices, which operate under the same brand name but as an individual investment management companies such as HS Malta plc, HS Spain, HS India Limited, HS Malaysia Limited. The firm definition should be broad enough to include all geographical offices under one umbrella. This is just a recommendation not a requirement.

Instructor's Note:

At Level I you are required to know the details of Provision 0 (Fundamentals of Compliance). For provisions 1 – 8 you just need to know the basic descriptions which are given below:

1. Input data

Consistency of input data used to calculate performance is critical to effective compliance with the GIPS standards and establishes the foundation for full, fair, and comparable investment performance presentations. For periods beginning on or after 1 January 2011, all portfolios must be valued in accordance with the definition of fair value and the GIPS valuation principles.

2. Calculation Methodology

Achieving comparability among investment management firm's performance presentations require uniformity in the methods used to calculate returns. The GIPS standards mandate the use of certain calculation methodologies to facilitate comparability.

3. Composite Construction

A composite is an aggregation of one or more portfolios managed according to a similar investment mandate, objective, or strategy. The composite return is the asset-weighted average of the performance of all portfolios in the composite. Creating meaningful composites is essential to the fair presentation, consistency, and comparability of performance over time and among firms. Assume there are two portfolios in a composite: portfolio 1 with a value of \$10 million and portfolio 2 with a value of \$90 million. The returns of the two portfolios are 10% and 12% respectively. The overall return of the composite must be closer to 12% as the 12% return has a 90% weightage.

4. Disclosure

Disclosure allows firms to elaborate on the data provided in the presentation and give the reader the proper context in which to understand the performance. To comply with the GIPS standards, firms must disclose certain information in all compliant presentations regarding their performance and the policies adopted by the firm. One of the essential disclosures for every firm is the claim of compliance. Once a firm meets all the requirements of the GIPS

standards, it must appropriately use the claim of compliance to indicate compliance with the GIPS standards. The allowed format for firms that claim compliance is: <<Name of firm>> claims compliance with the Global Investment Performance Standards (GIPS) and has prepared and presented this report in compliance with the GIPS Standards.

5. Presentation and Reporting

After constructing the composites, gathering the input data, calculating returns, and determining the necessary disclosures, the firm must incorporate this information in presentations based on the requirements in the GIPS standards for presenting investment performance. No finite set of requirements can cover all potential situations or anticipate future developments in investment industry structure, technology, products, or practices. When appropriate, firms have the responsibility to include in GIPS-compliant presentations information not addressed by the GIPS standards.

6. Real Estate

Unless otherwise noted, this section supplements all of the required and recommended provisions in Sections 0-5. Real estate provisions were first included in the 2005 edition of the GIPS standards and became effective 1 January, 2006. The 2010 edition of the GIPS standards includes new provisions for closed-end real estate funds. Firms should note that certain provisions of sections 0-5 do not apply to real estate investments or are superseded by the provisions within section 6. The provisions that do not apply have been noted within section 6.

7. Private Equity

Unless otherwise noted, this section supplements all of the required and recommended provisions in sections 0-5. Private equity provisions were first included in the 2005 edition of the GIPS standards and became effective 1 January, 2006. Firms should note that certain provisions in sections 0-5 do not apply to private equity investments or are superseded by the provisions within section 7. The provisions that do not apply have been noted within section 7.

8. Wrap Fee/Separately Managed Account (SMA) Portfolios

Unless otherwise noted, this section supplements all of the required and recommended provisions in sections 0-5. Firms should note that certain provisions in sections 0-5 of the GIPS standards do not apply to wrap fee/SMA portfolios or are superseded by the provisions within section 8. The provisions that do not apply here have been noted within section 8.

Sample Presentation

A sample GIPS-compliant presentation report is presented below. Some of the important aspects that you can take note of are:

- Name of firm: Sample 1 Investment Firm; Composite: Balance Growth
- Gross return, net return, and no. of portfolios re required data which are presented.

Sample 1 Investment Firm Balanced Growth Composite *(from the curriculum)***1 January 2002 through 31 December 2011**

Year	Composite Gross Return (%)	Composite Net Return (%)	Custom Benchmark Return (%)	Composite 3-Yr St Dev (%)	Benchmark 3-Yr Sr Dev (%)	Number of Portfolios	Internal Dispersion (%)	Composite Assets (\$ M)	Firm Assets (\$ M)
2002	-10.5	-11.4	-11.8			31	4.5	165	236
2003	16.3	15.1	13.2			34	2.0	235	346
2004	7.5	6.4	8.9			38	5.7	344	529
2005	1.8	0.8	0.3			45	2.8	445	695
2006	11.2	10.1	12.2			48	3.1	520	839
2007	6.1	5.0	7.1			49	2.8	505	1,014
2008	-21.3	-22.1	-24.9			44	2.9	475	964
2009	16.5	15.3	14.7			47	3.1	493	983
2010	10.6	9.5	13.0			51	3.5	549	1,114
2011	2.7	1.7	0.4	7.1	7.4	54	2.5	575	1,236

Sample 1 Investment Firm claims compliance with the Global Investment Performance Standards (GIPS®) and has prepared and presented this report in compliance with the GIPS standards. Sample 1 Investment Firm has been independently verified for the periods 1 January 2000 through 31 December 2010. The verification report is available upon request. Verification assesses whether (1) the firm has complied with all the composite construction requirements of the GIPS standards on a firm-wide basis and (2) the firm's policies and procedures are designed to calculate and present performance in compliance with the GIPS standards. Verification does not ensure the accuracy of any specific composite presentation.

Notes:

1. Sample 1 Investment Firm is a balanced portfolio investment manager that invests solely in US-based securities. Sample 1 Investment Firm is defined as an independent investment management firm that is not affiliated with any parent organization. Policies for valuing portfolios, calculating performance, and preparing compliant presentations are available upon request.
2. The Balanced Growth Composite includes all institutional balanced portfolios that invest in large-cap US equities and investment-grade bonds with the goal of providing long-term capital growth and steady income from a well-diversified strategy. Although the strategy allows for equity exposure ranging between 50–70%, the typical allocation is between 55–65%. The account minimum for the composite is \$5 million.

3. The custom benchmark is 60% YYY US Equity Index and 40% ZZZ US Aggregate Bond Index. The benchmark is rebalanced monthly.
4. Valuations are computed and performance is reported in US dollars.
5. Gross-of-fees returns are presented before management and custodial fees but after all trading expenses. Composite and benchmark returns are presented net of non-reclaimable withholding taxes. Net-of-fees returns are calculated by deducting the highest fee of 0.83% from the monthly gross composite return. The management fee schedule is as follows: 1.00% on the first \$25 million; 0.60% thereafter.
6. This composite was created in February 2000. A complete list of composite descriptions is available upon request.
7. Internal dispersion is calculated using the equal-weighted standard deviation of annual gross returns of those portfolios that were included in the composite for the entire year.
8. The three-year annualized standard deviation measures the variability of the composite and the benchmark returns over the preceding 36-month period.

Summary

LO.a: Describe the key features of the GIPS standards and the fundamentals of compliance.

Refer to sections 'Key Features of the GIPS Standards' and '0. Fundamentals of Compliance'.

LO.b: Describe the scope of the GIPS standards with respect to an investment firm's definition and historical performance record.

Investment Firm's Definition

- Firms must be defined as an investment firm, subsidiary or a division that is held out to clients and prospects as a 'distinct business entity'.
- If a firm has different geographic locations, all doing business under the same name, then the definition of the firm must include branches from all locations.

Historical Performance Record

Initially,

- A firm must present a minimum of five years of compliant presentation.
- If the firm or composite has been in existence for less than five years, the presentation should include performance since inception.

After initial compliance,

- The firm must add one year of compliant presentation each year,
- So that the firm eventually presents a minimum performance history of 10 years.

LO.c: Explain how the GIPS standards are implemented in countries with existing standards for performance reporting and describe the appropriate response when the GIPS standards and local regulations conflict.

- In countries where there are no investment performance regulations, use and promote the GIPS standard.
- In countries where there are existing laws and regulations regarding performance presentation, adhere to GIPS in addition to the local laws.
- In case of a conflict with the local laws, follow the local law but disclose the conflict

LO.d: Describe the nine major sections of the GIPS standards.

Refer to section 'Nine Major Sections of the GIPS Standards'.

Practice Questions

1. Which of the following is *not* a key feature of the GIPS standards?
 - A. All actual, discretionary, fee-paying and non-fee paying portfolios must be included in at least one composite.
 - B. Firms must use accurate input data, follow certain calculation methodologies and disclose the method used.
 - C. Firms must comply with all requirements of the GIPS standards.
2. If a composite has been in existence for less than five years, then for what period is the firm most likely required to present GIPS-compliant performance?
 - A. Two years.
 - B. One year.
 - C. Since inception date.
3. Eritba is a small island nation in the Pacific where laws and regulations exist for calculation and presentation of investment performance. According to the GIPS standards, what laws are firms in Eritba encouraged to comply with for the calculation and presentation of investment performance?
 - A. GIPS standards only.
 - B. Applicable law, regulations, and GIPS standards.
 - C. Applicable law and regulations only.
4. Which of the following is *not* one of the nine sections of the provisions of the Global Investment Performance Standards?
 - A. Disclosure.
 - B. Processed Data.
 - C. Real Estate.

Solutions

1. A is correct. It is not mandatory for non-fee paying portfolios to be included in at least one composite. Section 3. LO.a.
2. C is correct. If a firm has been in existence for less than five years, then a firm is required to present compliant presentation since the firm's inception date, or the composite inception date. Section 4. LO.b.
3. B is correct. Firms are encouraged to comply with applicable laws, regulatory requirements in addition to the GIPS Standards. This is because compliance with applicable law does not imply compliance with the GIPS Standards. Section 6. LO.c.
4. B is correct. 'Input Data', not 'Processed Data' is a section of the provisions of the GIPS standards. Section 7. LO.d.

R06 Time Value of Money

Introductory Note

Financial Calculator: CFA Institute allows only two calculator models during the exam:

- Texas Instruments BA II Plus (including BA II Plus Professional) and
- Hewlett Packard 12C (including the HP 12C Platinum, 12C Platinum 25th anniversary edition, 12C 30th anniversary edition, and HP 12C Prestige)

Unless you are already comfortable with the HP financial calculator, we recommend using the Texas Instruments financial calculator. Explanations and keystrokes in our study materials are based on the Texas Instruments BA II Plus calculator.

Before you start using the calculator to solve problems, we recommend that you set the number of decimal places to 'floating decimal'.

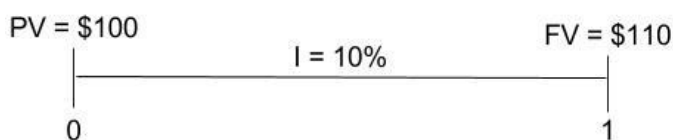
1. Introduction

If you have \$100 today, versus an option to receive \$100 after three years, what would you prefer?



Obviously, you would prefer \$100 today. Even though you have the same amount (\$100) in both cases, you prefer \$100 today. This means that there has to be some value associated with time, because you are putting more value on the \$100 that you are getting today, relative to the \$100 at a later point in time. This is known as 'time value of money.'

Let us say that you are indifferent between \$100 dollars today versus \$ 110 after one year.



Present value (PV): The money today or the value today is called the present value (PV = 100). This could be an investment which you make at time 0.

Future value (FV): The value at a future point in time is called the future value (FV = 110).

Interest rate (I): The relationship or the link between present value and future value is established through an interest rate (I = 10%).

In this reading, we are essentially going to talk about these concepts: present value (PV), future value (FV), and the way we link these two concepts using interest rates (I).

2. Interest Rates: Interpretation

Let's discuss the different interpretations of interest rates using an example. Say you lend \$900 today and receive \$990 after one year (-ve sign indicates outflow).



Interest rates can be interpreted as:

1. **Required rate of return:** The fact that you are willing to give \$900 today on the condition that you get \$990 after one year means that to engage in this transaction, you require a return of 10%. (Simple calculation will show you that the interest rate in this transaction is 10%).
2. **Discount rate:** You can discount the money that you will receive after one year i.e. \$990 at 10% to get the present value of \$900 ($990/1.1 = 900$). Therefore, the 10% can also be thought of as a discount rate.
3. **Opportunity cost:** Let's say instead of lending the \$900, you spent it on something else. You have then forgone the opportunity to earn 10% interest. Therefore, 10% can also be thought of as an opportunity cost.

Interest Rates: Investor Perspective

As an investor, we can think of interest rate as a sum of the following components:

$$\text{Interest rate} = \text{Real risk-free interest rate} + \text{Inflation premium} + \text{Default risk premium} + \text{Liquidity premium} + \text{Maturity premium}$$

Let's look at the different components.

- **Real risk-free interest rate:** This is the rate that you get on a security that has no risk and is extremely liquid. We make an assumption here that there is no inflation.
- **Inflation premium:** We can then add on an inflation premium. Inflation premium is the expected annual inflation in the upcoming period.
- **Default risk premium:** We can also then add a default risk premium. This is the additional premium that investors require because of the risk of default.
Example: Let's say that you lend \$100 each to person A and person B. However, B has a high risk of default, so you are worried that he might not pay. Therefore you might demand a higher return from B as compared to A, because of the risk of default. This additional return that you demand is called the default risk premium.
- **Liquidity premium:** Next we have liquidity premium. This is the premium that investors demand because of the lack of liquidity of an investment.

Example: Think of two investments C and D which are similar in all regards. The only difference is that investment C is extremely liquid, whereas investment D is not that liquid. Clearly as investors, we will demand a higher return on D because it is not easy to sell. This additional return that we demand is called the liquidity premium.

- **Maturity premium:** Finally we have the maturity premium. This is the premium that investors demand on a security with long maturity.

Example: Let's say we have two securities E and F. Security E has a maturity of 1 year and security F has a maturity of 4 years. Because of the longer maturity, F has more risk, in terms of its price being more sensitive to changes in interest rate.

Instructor's Note: You will understand this concept better when you study fixed income securities. But for now, you can take it as a given that F has higher risk because of the longer maturity.

Obviously, investors will demand some compensation for the higher level of risk. This additional return that investors demand is called the maturity premium.

Nominal risk free rate:

Nominal risk free rate = Real risk-free interest rate + Inflation premium.

So if the real risk free rate is 3% and the inflation premium is 2%, then the nominal risk free rate is 5%.

Instructor's Note: On the exam if you get a term 'risk-free rate' with no mention of whether the rate is real or nominal, then the assumption is that we are talking about the nominal risk-free rate.

Example

<i>Investments</i>	<i>Maturity (in years)</i>	<i>Liquidity</i>	<i>Default risk</i>	<i>Interest Rates(%)</i>
A	1	High	Low	2.0
B	1	Low	Low	2.5
C	2	Low	Low	r
D	3	High	Low	3.0
E	3	Low	High	4.0

1. Explain the difference between the interest rates on Investment A and Investment B.
2. Estimate the default risk premium.
3. Calculate upper and lower limits for the interest rate on Investment C, r.

Solution:

1. Investments A and B have the same maturity and the same default risk. However B has a lower liquidity as compared to A. Hence investors will demand a liquidity premium on B. The difference between their interest rates i.e. $2.5 - 2.0 = 0.5\%$ is equal to the liquidity premium.

2. Consider investments D and E, they have the same maturity, but different liquidity and different default risk. Let's make liquidity the same and create a new low liquidity version of D. This version will have a higher interest rate, because now investors will demand a liquidity premium. We have already determined that the liquidity premium is 0.5%. Therefore, the low liquidity version of D will have an interest rate of $3.0 + 0.5 = 3.5\%$.

Now compare this version of D with investment E. The only difference between the two is default risk. E has a higher default risk. Therefore, the difference between their interest rates i.e. $4.0 - 3.5 = 0.5\%$ must be equal to the default risk premium.

3. Notice that between B and C, the only difference is that C has a longer maturity. Therefore, interest rate of C must be higher than B (2.5%).
Also notice that between C and the low liquidity version of D, the only difference is that C has a shorter maturity. Therefore, interest rate on C has to be lower than the low liquidity version of D (3.5%).
So the range for C is $2.5 < r < 3.5$.

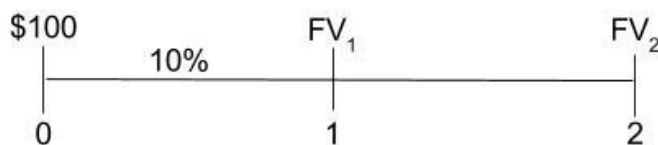
3. The Future Value of a Single Cash Flow

Let's understand this concept with a simple example.

Say present value (PV) = \$100 and interest rate (r) = 10%.

What is the future value (FV) after one year?

What is the future value (FV) after two years?



The future value of a single cash flow can be computed using the following formula:

$$FV_N = PV (1 + r)^N$$

where:

FV_N = future value of the investment

N = number of periods

PV = present value of the investment

r = rate of interest

Therefore,

$$FV_1 = 100 (1 + 0.1)^1 = \$110$$

$$FV_2 = 100 (1 + 0.1)^2 = \$121$$

Notice that with compound interest, after two years we have \$121. Whereas, with simple interest, after two years we would have \$120. The difference between the two values (\$1)

represents the interest on interest component. In Year 2, we not only receive interest on the \$100 principal, but we also receive interest on the \$10 interest earned in Year 1 that has been reinvested.

Example

Cyndia Rogers deposits \$5 million in her savings account. The account holders are entitled to a 5% interest. If Cyndia withdraws cash after 2.5 years, how much cash would she *most likely* be able to withdraw?

Solution:

$$FV_N = PV (1 + r)^N$$

$$FV_{2.5} = 5 (1 + 0.05)^{2.5} = \$5.649 \text{ million}$$

FV Calculation using a Financial Calculator

You will often use the following keys on your TI BA II Plus calculator:

N = number of periods

I/Y = rate per period

PV = present value

FV = future value

PMT = payment

CPT = compute

One important point to note is the signs used for PV and FV. If the value for PV is negative “-”, then the value for FV is positive “+”. An inflow is often represented as a positive number, while outflows are denoted by negative numbers.

Before you begin set the number of decimal points on your calculator to 9 to increase accuracy.

Keystrokes	Explanation	Display
[2nd] [FORMAT] [ENTER]	Get into format mode	DEC = 9
[2nd] [QUIT]	Return to standard calc mode	0

Question: You invest \$100 today at 10% compounded annually. How much will you have in 5 years?

The key strokes to compute the future value of a single cash flow are illustrated below.

Keystrokes	Explanation	Display
[2nd] [QUIT]	Return to standard calc mode	0
[2nd] [CLR TVM]	Clears TVM Worksheet	0
5 [N]	Five years/periods	N = 5

10 [I/Y]	Set interest rate	I/Y = 10
100 [PV]	Set present value	PV = 100
0 [PMT]	Set payment	PMT = 0
[CPT] [FV]	Compute future value	FV = -161.05

3.1. The Frequency of Compounding

When our compounding frequency is not annual, we use the following formula to compute future value:

$$FV_N = PV \left(1 + \frac{r_s}{m} \right)^{mN}$$

where:

r_s = the stated annual interest rate in decimal format

m = the number of compounding periods per year

N = the number of years

Let's understand this concept using an example.

You invest \$80,000 in a 3-year certificate of deposit. This CD offers a stated annual interest rate of 10% compounded quarterly. How much will you have at the end of three years?

Solution:

There are two methods to solve this question.

Formula Method

PV is \$80,000.

The stated annual rate is 10%.

The number of compounding periods per year is 4. The total number of periods is $4 \times 3 = 12$.

Therefore future value after 12 quarters (3 years) is

$$FV_{12} = \$80,000 (1 + 0.025)^{12} = \$107,591$$

Calculator Method

You can also solve this problem using a financial calculator; the key strokes are given below:

$N = 12$, $I/Y = 2.5\%$, $PV = \$80,000$, $PMT = 0$, $CPT FV = -\$107,591$

PMT is 0 because there are no intermediate payments in this example.

Example

Donald invested \$3 million in an American bank that promises to pay 4% compounded daily.

Which of the following is *closest* to the amount Donald receives at the end of the first year?

Assume 365 days in a year.

- A. \$3.003 million
- B. \$3.122 million
- C. \$3.562 million

Solution

The correct answer is B.

Formula Method

$$FV_N = PV \left(1 + \frac{r_s}{m} \right)^{mN}$$

$$FV_{1 \text{ year}} = 3 \text{ million} \left(1 + \frac{0.04}{365} \right)^{365} = \$3.122 \text{ million}$$

Calculator Method

N = 365, I/Y = 4/365%, PV = \$3 million, PMT = 0; CPT FV = -\$3.122 million

3.2. Continuous Compounding

We saw examples with annual compounding. Then we discussed quarterly compounding and in the above example we looked at daily compounding. If we keep increasing the number of compounding periods until we have infinite number of compounding periods per year, then we can say that we have continuous compounding.

The formula for computing future values with continuous compounding is:

$$FV_N = PVe^{rN}$$

where:

r = continuously compounded rate

N = the number of years

Let's look at an example.

An investment worth \$50,000 earns interest that is compounded continuously. The stated annual interest is 3.6%. What is the future value of the investment after 3 years?

Solution:

PV = \$50,000; r = 0.036; N = 3

$FV = 50,000 e^{0.036 \times 3} = \$55,702$

Concept Building Exercise

Assume that the stated annual interest rate is 12%. What is the future value of \$100 at different compounding frequencies? What is the return?

Frequency	Future value of \$100	Return
Annual	112.00	12.00%
Semiannual	112.36	12.36%
Quarterly	112.55	12.55%
Monthly	112.68	12.68%
Daily	112.75	12.75%
Continuous	112.75	12.75%

Instructor's Note:

1. For the same stated annual rate, the returns keep getting better as we compound more often.

2. If you have two banks that offer the following rates

A: 12.5% compounded annually

B: 12% compounded daily

Which bank is better?

Even though A's 12.5% looks better, B's 12% compounded daily will effectively give you a return of 12.75%. Therefore the offer from bank B is better.

3.3. Stated and Effective Rates

Now we come to the related concept of stated versus effective rates. In the above concept building exercise, the stated rate was 12% across the board, but the effective rate that an investor actually earns depends on the compounding frequency. The effective rates were different for different compounding frequencies.

If we are given a compounding frequency, we can compute effective rates using the following formulae:

Effective annual rate for discrete compounding:

$$\text{EAR} = (1 + \text{periodic interest rate})^m - 1$$

where:

m = number of compounding periods in one year

- For daily compounding, m = 365
- For monthly compounding, m = 12
- For quarterly compounding, m = 4
- For semiannual compounding, m = 2

For example, for a stated annual rate of 12% and quarterly compounding, the EAR will be equal to:

$$\text{EAR} = (1 + 0.12/4)^4 - 1 = 1.1255 - 1 = 0.1255 = 12.55\%$$

Instructor's Note:

A lot of people get confused about the -1 at the end of the formula. The idea is actually fairly straight forward. Basically we have 1.03^4 which is telling us how much \$1 will become at the end of 4 periods. \$1 is going to become \$1.1255. But this is not a rate. To figure out the rate we have to subtract the original \$1 that we invested. So we are left with 0.1255 which is our effective rate.

Effective annual rate for continuous compounding:

$$\text{EAR} = e^{r_s} - 1$$

where r_s = stated annual interest rate

For example, for a stated annual rate of 12% and continuous compounding, the EAR will be equal to:

$$\text{EAR} = e^{0.12} - 1 = 1.1275 - 1 = 0.1275 = 12.75\%$$

4. The Future Value of a Series of Cash Flows

The different types of cash flows based on the time periods over which they occur include:

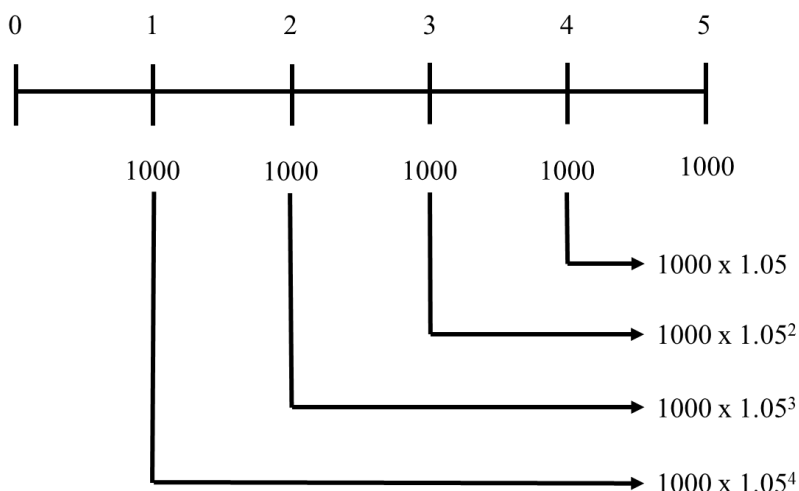
- **Annuity:** A finite set of equal sequential cash flows. The two types are
 - **Ordinary annuity:** The first cash flow occurs one period from now (indexed at $t = 1$).
 - **Annuity due:** The first cash flow occurs immediately (indexed at $t = 0$).
- **Perpetuity:** A set of equal never-ending sequential cash flows with the first cash flow occurring one period from today. (The period is finite in case of an annuity whereas in perpetuity it is infinite.)

4.1. Equal Cash Flows – Ordinary Annuity

Let's say that we have an ordinary annuity with $A = \$1,000$, $r = 5\%$ and $N = 5$, i.e. we are going to receive a payment of \$1,000 at the end of each year for the next 5 years and our discount rate is 5%. We can compute the FV of this annuity using the following three methods:

Brute-Force Method

We can take each individual cash flow and see how much each of these will be worth at the end of five years. Then we can add all the values to compute the total FV of the annuity.



For instance, the first \$1,000 deposit made at $t = 1$ will compound over four periods; the second deposit of \$1,000 will compound over three periods and so on. We then add the future values of all payments to arrive at the future value of the annuity which is \$5,525.63.

Formula Method

The future value of an annuity can also be computed using the following formula:

$$FV_N = A \left[\frac{(1 + r)^N - 1}{r} \right]$$

where:

A = annuity amount

N = number of years

The term in square brackets is known as the 'future value annuity factor (FVAF)'. This factor gives the future value of an ordinary annuity of \$1 per period. Hence the formula given above can also be written as: $FV = A \times FVAF$.

Therefore, using the formula:

$$FV_5 = 1,000 \left[\frac{(1.05)^5 - 1}{0.05} \right] = \$5,525.63$$

Calculator Method

Given below are the keystrokes for computing the future value of an ordinary annuity.

Keystrokes	Explanation	Display
[2nd] [QUIT]	Return to standard calc mode	0
[2 nd] [CLR TVM]	Clears TVM Worksheet	0
5 [N]	Five years/periods	N = 5
5 [I/Y]	Set interest rate	I/Y = 5
0 [PV]	0 because there is no initial investment	PV = 0
1000 [PMT]	Set annuity payment	PMT = 1000
[CPT] [FV]	Compute future value	FV = -5525.63

On the exam you should use the calculator method, because this is the fastest method and does not require you to memorize the annuity formula.

Example

Haley deposits \$24,000 in her bank account at the end of every year. The account earns 12% per annum. If she continues this practice, how much money will she have at the end of 15 years?

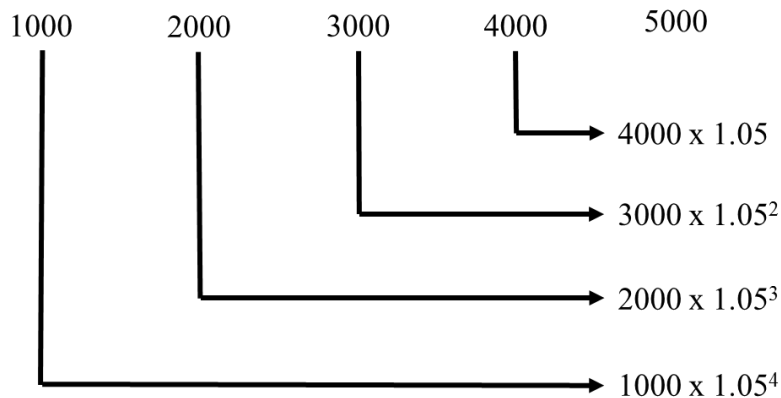
Solution:

N = 15, I/Y = 12%, PV = 0, PMT = \$24,000; CPT → FV = -\$894,713.15

4.2. Unequal Cash Flows

If cash flow streams are unequal, the future value annuity factor cannot be used. In this case, we find the future value of a series of unequal cash flows by compounding the cash flows one at a time.

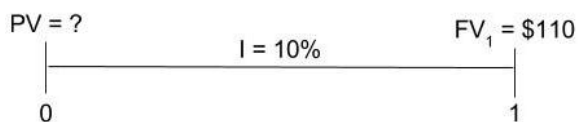
This concept is illustrated in the figure below. We need to find the future value of five cash flows: \$1,000 at the end of Year 1; \$2,000 at the end of Year 2; \$3,000 at the end of Year 3; \$4,000 at the end of Year 4; and \$5,000 at the end of Year 5.



The future value is $\$5,000 + \$4,000 \times 1.05 + \$3,000 \times 1.05^2 + \$2,000 \times 1.05^3 + \$1,000 \times 1.05^4 = \$16,038$.

5. The Present Value of a Single Cash Flow

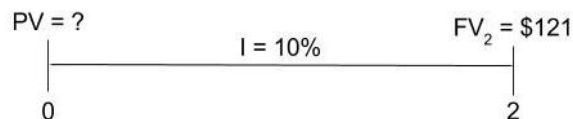
Let's say that one year from today you will receive a cash flow of \$110. What is the value of that \$110 today? (Assume that the interest rate is 10%)



$$PV = \frac{FV_1}{(1 + 0.1)^1} = \frac{\$110}{(1 + 0.1)^1} = \$100$$

The \$110 one year from today has a present value of \$100. In other words, you will be indifferent between \$100 today and \$110 one year from today.

What if you were going to receive \$121 at the end of two years, what is its present value?



$$PV = \frac{FV_2}{(1 + 0.1)^2} = \frac{\$121}{(1 + 0.1)^2} = \$100$$

Using these two examples we can write a general formula for computing PV. Given a cash flow that is to be received in N periods and an interest rate of r per period, the PV can be computed as:

$$PV = \frac{FV_N}{(1 + r)^N}$$

where:

N = number of periods

r = rate of interest

FV = future value of investment

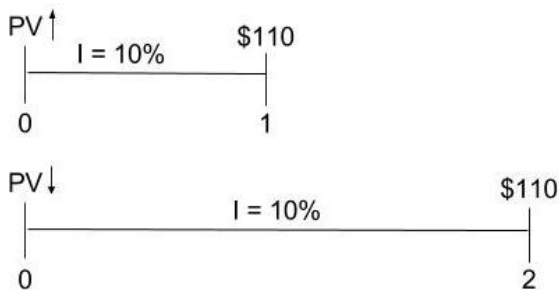
Instructor's Note

Notice that this formula can also be obtained by simply rearranging the formula for FV that we studied earlier.

$$FV_N = PV (1 + r)^N \rightarrow PV = \frac{FV_N}{(1+r)^N}$$

Based on this formula we can make two observations

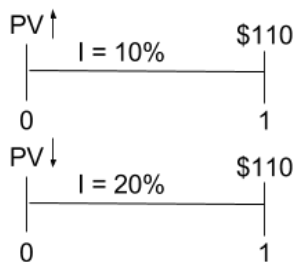
1. For a given discount rate, the farther in the future the amount to be received, the smaller the amount's present value.



Mathematical explanation: In the first case the PV is $\$110/1.1$, whereas in the second case PV is $\$110/1.1^2$. Since we are dividing by a larger number the PV will be lower in the second case.

Intuitive explanation: Clearly receiving a certain amount of money sooner is better than receiving the same amount of money latter.

2. Holding time constant, the larger the discount rate, the smaller the present value of a future amount.



In the first case PV is \$110/1.1, whereas in the second case PV is \$110/1.2. Clearly the PV is going to be lower in the second case, because we are dividing by a larger number.

Example

Liam purchases a contract from an insurance company. The contract promises to pay \$600,000 after 8 years with a 5% return. What amount of money should Liam most likely invest? Solve using the formula and TVM functions on the calculator.

Solution:

Formula Method

$$PV = \frac{FV_N}{(1 + r)^N} = \frac{\$600,000}{(1.05)^8} = \$406,104$$

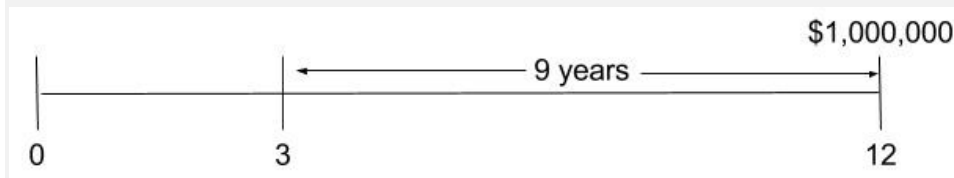
Calculator Method

N = 8, I/Y = 5%, PMT = 0, FV = \$600,000; CPT PV → PV = - \$406,104

Example

Mathews wishes to fund his son, Nathan's, college tuition fee. He purchases a security that will pay \$1,000,000 in 12 years. Nathan's college begins 3 years from now. Given that the discount rate is 7.5%, what is the security's value at the time of Nathan's admission?

Solution:



$$PV_3 = \frac{\$1,000,000}{(1.075)^9} = \$521,583.47$$

Example

Orlando is a manager at an Australian pension fund. 5 years from today he wants a lump sum amount of AUD40,000. Given that the current interest rate is 4% a year, compounded monthly, how much should Orlando invest today?

Solution:

We have monthly compounding, therefore the inputs to our calculator will be

$$N = 5 \times 12 = 60$$

$$I = 4 / 12\%$$

$$PMT = 0$$

$$FV = \$40,000$$

$$CPT PV = - \$32,760.12$$

6. The Present Value of a Series of Cash Flows

6.1. The Present Value of a Series of Equal Cash Flows

Ordinary Annuity

An ordinary annuity is a series of equal payments at equal intervals for a finite period of time. Examples of ordinary annuity: mortgage payments, pension income etc.

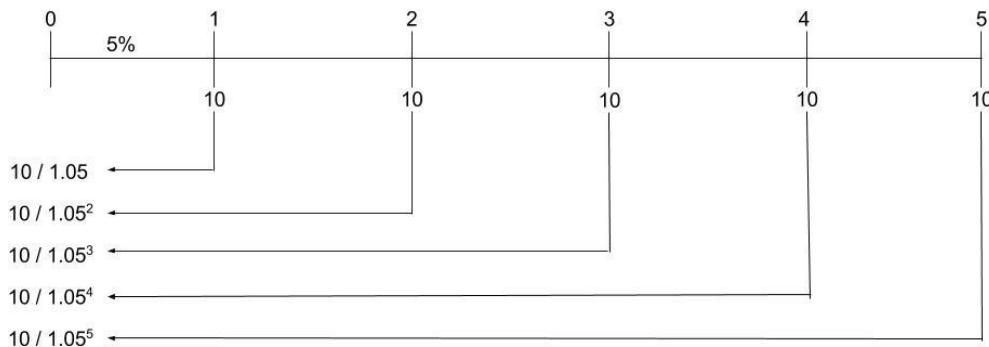
Consider an ordinary annuity with $A = \$10$, $r = 5\%$ and $N = 5$.



We can compute the PV of this annuity using three methods

Brute-Force Method:

Simply take each cash flow and compute the PV. Add all values to get the PV for the annuity.



PV = \$43.29.

Formula Method:

The present value can also be computed using the following formula:

$$PV = A \left[\frac{1 - \left(\frac{1}{(1+r)^N} \right)}{r} \right]$$

where:

A = annuity amount

r = interest rate per period corresponding to the frequency of annuity payments

N = number of annuity payments

The term in square brackets is called the present value annuity factor (PVAF). Hence the equation above can also be written as: $PV = A \times PVAF$.

Therefore, using the formula we get,

$$PV = 10 \left[\frac{1 - \left(\frac{1}{(1+0.05)^5} \right)}{0.05} \right] = \$43.29$$

Calculator Method:

The keystrokes to solve this using a financial calculator are given below:

Keystrokes	Explanation	Display
[2nd] [QUIT]	Return to standard calc mode	0
[2nd] [CLR TVM]	Clears TVM Worksheet	0
5 [N]	Five years/periods	N = 5
5 [I/Y]	Set interest rate	I/Y = 5
0 [FV]	Set to 0 because there is no final payment other than the periodic annuity amounts	FV = 0
10 [PMT]	Set annuity payment	PMT = 10
[CPT] [PV]	Compute present value	PV = -43.29

Annuity Due

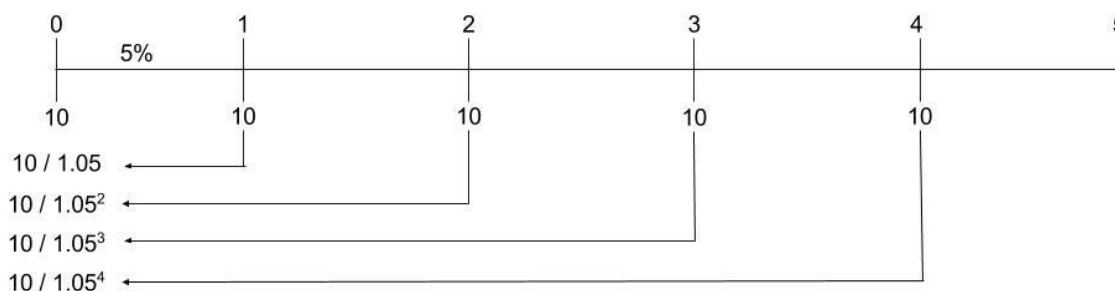
With an annuity due the first payment is received at the start of the first period. So if we have an annuity due with $A = \$10$, $r = 5\%$ and $N = 5$. The cash flows will be:



Again there are three methods to calculate the PV of this annuity due.

Brute-Force method

Take each cash flow and compute the PV. Add all values to get the PV for the annuity.



PV = \$45.46.

Notice that with an annuity due you are receiving money faster, which means that the

PV annuity due (\$45.46) > PV ordinary annuity (\$43.29).

Formula method:

We can also use the following formula:

$$PV = A \left[\frac{1 - \frac{1}{(1+r)^N}}{r} \right] (1 + r)$$

where:

A = annuity amount

r = interest rate per period corresponding to the frequency of annuity payments

N = number of annuity payments

Therefore using the formula,

$$PV = 10 \left[\frac{1 - \frac{1}{(1+0.05)^5}}{0.05} \right] (1 + 0.05) = \$45.46$$

Instructor's Note:

Notice that $A \left[\frac{1 - \frac{1}{(1+r)^N}}{r} \right]$ is basically the formula for computing the PV of an ordinary annuity.

If you use the ordinary annuity formula the PV that you get will be at time period -1. So this needs to be taken forward one period by multiplying it by $(1 + r)$

Calculator Method:

Set the calculator to BGN mode. This tells the calculator that payments happen at the start of every period. (The default calculator setting is END mode which means that payments happen at the end of every period). The keystrokes are shown below:

Keystrokes	Explanation	Display
[2nd] [BGN] [2nd] [SET]	Set payments to be received at beginning rather than end	BGN
[2nd] [QUIT]	Return to standard calc mode	BGN 0
[2nd] [CLR TVM]	Clears TVM Worksheet	BGN 0
5 [N]	Four years/periods	BGN N = 5
5 [I/Y]	Set interest rate	BGN I/Y = 5
10 [PMT]	Set payment	BGN PMT = 10
0 [FV]	Set future value	BGN FV = 0
[CPT] [PV]	Compute present value	BGN PV = -45.46

[2nd] [BGN] [2nd] [SET]	Set payments to be received at the end	END
[2nd] [QUIT]	Return to standard calc mode	0

Always remember to put your calculator back in the END mode after you are done with the calculations.

6.2. The Present Value of an Infinite Series of Equal Cash Flows – Perpetuity

A perpetuity is a series of never ending equal cash flows. The present value of perpetuity can be calculated by using the following formula:

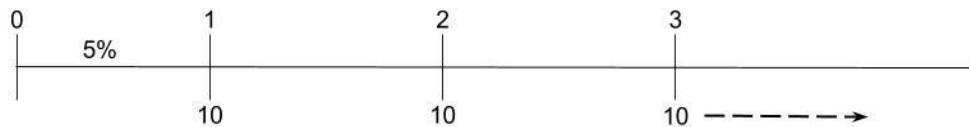
$$PV = \frac{A}{r}$$

where:

A = annuity amount

r = discount rate

Let's say that we have a really simple perpetuity where we receive \$10 at the end of every year forever, and let's say that the interest rate is 5%.



The PV of this perpetuity can be computed as:

$$PV = \frac{10}{0.05} = \$200$$

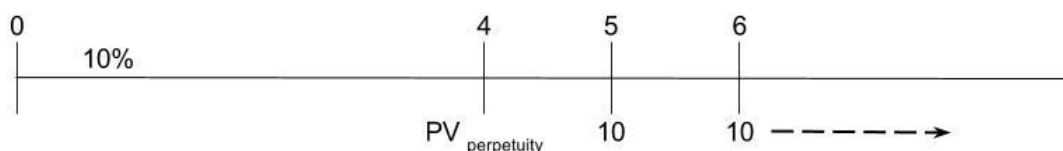
Instructor's Note:

Keep in mind that the present value of \$200 is one period before the first cash flow. Many students show the present value of \$200 at the same time as the first cash flow, which is incorrect.

6.3. Present Values Indexed at Times Other Than t=0

An annuity or perpetuity beginning sometime in the future can be expressed in present value terms one period prior to the first payment. That value can then be discounted back to today's present value.

Let's say you are offered a cash flow of \$10 at the end of year 5, end of year 6 and so on forever. What is the PV of these cash flows, assuming a discount rate of 10%?



The PV of the perpetuity at the end of year 4 can be computed as:

$$PV_4 = \frac{10}{0.1} = \$100$$

This value has to be discounted back four periods to get the PV at time period 0.

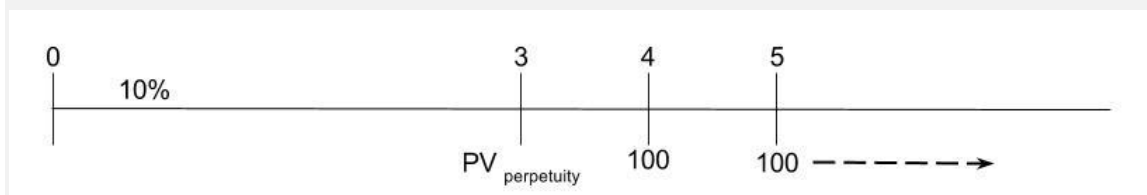
$$PV_0 = \$100 / 1.1^4 = \$68.30$$

Example

Bill Graham is willing to pay for a perpetual preferred stock that pays dividends worth \$100 per year indefinitely. The first payment will be received at $t = 4$. Given that the required rate of return is 10%, how much should Mr. Graham pay today?

Solution:

The time line for this scenario is



The PV of the perpetuity at the end of year 3 can be computed as

$$PV_3 = \frac{100}{0.1} = \$1,000$$

This value has to be discounted back 3 periods to get the PV at time period 0.

$$PV_0 = \$1,000 / 1.1^3 = \$751.31$$

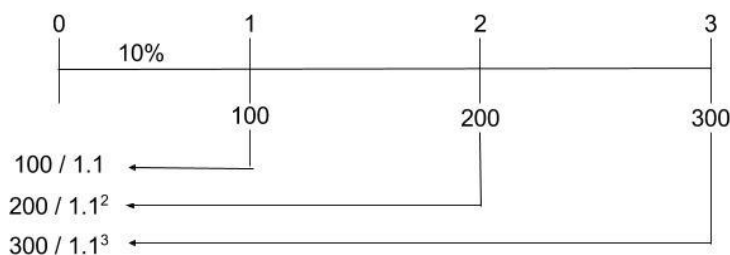
6.4. The Present Value of a Series of Unequal Cash Flows

When we have unequal cash flows, we can first find the present value of each individual cash flow and then sum the respective present values.

Let's say that we have the following cash flows:

<i>Time period</i>	<i>Cash Flow</i>
1	\$100
2	\$200
3	\$300

The PV of these cash flows can be computed as:



$$PV = 100 / 1.1 + 200 / 1.1^2 + 300 / 1.1^3 = \$481.59$$

Example

Andy makes an investment with the expected cash flow shown in the table below. Assuming a discount rate of 9% what is the present value of this investment?

Time Period	Cash Flow(\$)
1	50
2	100
3	150
4	200
5	250

Solution:

$$PV = 50/1.09 + 100/1.09^2 + 150/1.09^3 + 200/1.09^4 + 250/1.09^5 = \$550.03$$

We can also use the cash flow register on our financial calculator to solve this problem quickly. The key strokes are as follows:

Keystrokes	Explanation	Display
[2nd] [QUIT]	Return to standard mode	0
[CF] [2nd] [CLR WRK]	Clear CF Register	CF = 0
0 [ENTER]	No cash flow at t = 0	CF0 = 0
[↓] 50 [ENTER]	Enter CF at t = 1	C01 = 50
[↓] [↓] 100 [ENTER]	Enter CF at t = 2	C02 = 100
[↓] [↓] 150 [ENTER]	Enter CF at t = 3	C03 = 150
[↓] [↓] 200 [ENTER]	Enter CF at t = 4	C04 = 200
[↓] [↓] 250 [ENTER]	Enter CF at t = 5	C05 = 250
[↓] [NPV] [9] [ENTER]	Enter discount rate	I = 9
[↓] [CPT]	Compute NPV	550.03

7. Solving for Rates, Number of Periods, or Size of Annuity Payments**7.1. Solving for Interest Rates and Growth Rates**

An interest rate can also be considered a growth rate. We can compute the rate using the formula method or the calculator method.

Example:

A \$100 deposit today grows to \$121 in 2 years. What is the interest rate? Use both the formula and the calculator method.

Using the formula $100(1+r)^2 = 121$. Therefore $r = 0.1$ or 10%

Using the calculator method, inputs to the calculator are

PV = -\$100 (When we enter both PV and FV, they should be given opposite signs to avoid a calculator error.)

$FV = \$121$
 $N = 2$
 $PMT = 0$
 $CPT I \rightarrow I = 10\%$

Example:

The population of a small town is 100,000 on 1 Jan 2000. On 31 December 2001 the population is 121,000. What is the growth rate?

Inputs to the calculator are

$PV = -\$100,000$
 $FV = \$121,000$
 $N = 2$
 $PMT = 0$
 $CPT I \rightarrow I = 10\%$

Example:

You invest \$900 today and receive a \$100 coupon payment at the end of every year for 5 years. In addition, you receive \$1,000 at the end of year 5. What is the interest rate?

Inputs to the calculator are

$PV = -\$900$
 $FV = \$1,000$
 $N = 5$
 $PMT = 100$
 $CPT I \rightarrow I = 12.83\%$

7.2. Solving for the Number of Periods

Similarly, we can determine the number of periods given other information such as future value, present value and interest rate.

Example:

You invest \$2,500. How many years will it take to triple the amount given that the interest rate is 6% per annum compounded annually? Use both the formula and the calculator method.

Formula Method:

$$FV = PV(1 + r)^N$$

$$7,500 = 2,500(1 + 0.06)^N$$

$$1.06^N = 3$$

$$N \times \ln 1.06 = \ln 3$$

$$N = \left(\frac{\ln 3}{\ln 1.06} \right) = 18.85$$

Calculator Method:

Using the calculator: $I/Y = 6\%$, $PV = \$2,500$, $PMT = 0$, $FV = -\$7,500$, $CPT N = 18.85$.

7.3. Solving for the Size of Annuity Payments

Given the number of periods, interest rate per period, present value and future value, it is easy to solve for the annuity payment amount. This concept can be applied to mortgages and retirement planning. Consider the following example.

Example:

Freddie bought a car worth \$42,000 today. He was required to make a 15% down payment. The remainder was to be paid as a monthly payment over the next 12 months with the first payment due at $t = 1$. Given that the interest rate is 8% per annum compounded monthly, what is the approximate monthly payment?

Loan amount = 85% of \$42,000 = $0.85 \times 42,000 = \$35,700$

PV = \$35,700

N = 12

I/Y = 8/12%

FV = 0

CPT PMT → PMT = \$3,105.48

7.4. Review of Present and Future Value Equivalence

Let's say that we have an ordinary annuity with $A = 10$, $r = 5\%$ and $N = 5$.



As per our discussion so far, we can compute the PV and FV of this annuity

PV (at time 0) = \$43.29 and FV (at time 5) = \$55.26

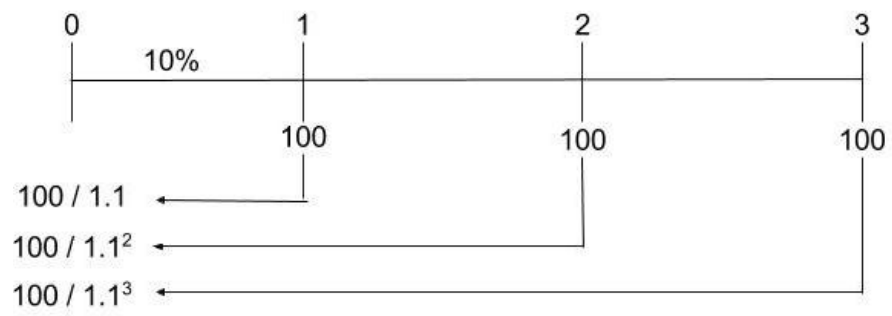
According to the concept of present and future value equivalence, a lump sum of \$43.29 at time 0 is equivalent to an annuity of \$10 over five years. Further, both these options are equivalent to a lump sum of \$55.26 at time 5. Given an interest rate of 5%, you would be indifferent between these choices.

7.5. The Cash Flow Additivity Principle

Amounts of money indexed at the same point in time are additive. For example, if you have the following cash flows:



You cannot simply add these three numbers. You have to take each of these numbers and bring them to a particular point in time. Let's say that we find the present values at time zero for each of these cash flows. According to this principle, these present values that are all indexed to time zero can be added.



Summary

LO.a: Interpret interest rates as required rates of return, discount rates, or opportunity costs.

An interest rate is the required rate of return. If you invest \$100 today on the condition that you get \$110 after one year, the required rate of return is 10%.

If the future value (FV) at the end of Year 1 is \$110, you can discount at 10% to get the present value (PV) of \$100. Hence, 10% can also be thought of as a discount rate.

Finally, if you spent \$100 on taking your spouse out for dinner you gave up the opportunity to earn 10%. Thus, 10% can also be interpreted as an opportunity cost.

LO.b: Explain an interest rate as the sum of a real risk-free rate, and premiums that compensate investors for bearing distinct types of risk.

Interest rate = Real risk-free interest rate + Inflation premium + Default risk premium + Liquidity premium + Maturity premium.

Nominal risk free rate = real risk free rate + inflation premium

LO.c: Calculate and interpret the effective annual rate, given the stated annual interest rate and the frequency of compounding.

The stated annual interest rate is a quoted interest rate that does not account for compounding within the year. The effective annual rate (EAR) is the amount by which a unit of currency will grow in a year when we do consider compounding within the year.

Example: If the stated annual rate is 12% with monthly compounding, the periodic or monthly rate is 1%. Since \$1 invested at the start of the year will grow to $1.01^{12} = 1.1268$, the EAR is 12.68%.

LO.d: Solve time value of money problems for different frequencies of compounding.

When our compounding frequency is not annual, we use the following formula to compute future value:

$$FV_N = PV \left(1 + \frac{r_s}{m} \right)^{mN}$$

where:

r_s = the stated annual interest rate in decimal format

m = the number of compounding periods per year

N = the number of years

If we keep increasing the number of compounding periods until we have infinite number of compounding periods per year, then we can say that we have continuous compounding. The formula to compute future value is:

$$FV_N = PVe^{rN}$$

where:

r = continuously compounded rate
 N = the number of years

LO.e: Calculate and interpret the future value (FV) and present value (PV) of a single sum of money, an ordinary annuity, an annuity due, a perpetuity (PV only), and a series of unequal cash flows.

The future value and present value for a single sum of money can be calculated using the following formulae:

$$FV = PV (1 + r)^n \text{ and } PV = FV / (1 + r)^n$$

An **annuity** is a finite set of equal sequential cash flows occurring at equal intervals. There are two types of annuities:

- **Ordinary annuity:** Cash flows occur at the end of every period.
- **Annuity due:** Cash flows occur at the start of every period (hence, the Period 1 cash flow occurs immediately).

The future value of an ordinary annuity can be computed using the following formula:

$$FV_N = A \left[\frac{(1 + r)^N - 1}{r} \right]$$

where:

A = annuity amount

N = number of years

The present value of ordinary annuity can be computed using the following formula:

$$PV = A \left[\frac{1 - \left(\frac{1}{(1+r)^N} \right)}{r} \right]$$

where:

A = annuity amount

r = interest rate per period corresponding to the frequency of annuity payments

N = number of annuity payments

With an annuity due the first payment is received at the start of the first period. The formula to calculate present value of annuity due is as follows:

$$PV = A \left[\frac{1 - \frac{1}{(1+r)^N}}{r} \right] (1 + r)$$

where:

A = annuity amount

r = interest rate per period corresponding to the frequency of annuity payments

N = number of annuity payments

Alternatively, you may also use the TVM keys on the calculator instead of the formulas to

compute the present values and the future values of annuities.

A perpetuity is a series of never ending equal cash flows. The present value of perpetuity can be calculated by using the following formula:

$$PV = \frac{A}{r}$$

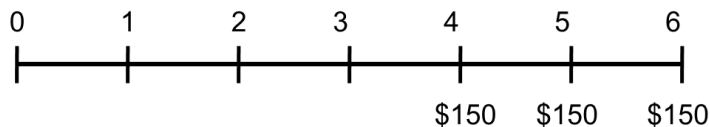
where:

A = annuity amount

r = discount rate

LO.f: Demonstrate the use of a time line in modeling and solving time value of money problems.

You can solve time value of money questions by showing cash flows on a timeline such as the one shown below:



Say you will receive \$150 at the end of Year 4, Year 5 and Year 6 and you want to calculate the PV at time 0. You can treat the three payments as an annuity and calculate the PV at the end of year 3. This value, assuming a 10% discount rate, is: \$373.03. We can then further discount \$373.03 to time 0. Plug: FV = \$373.03, N = 3, I = 10%, PMT = 0. Compute PV. You should get \$280.26.

Practice Questions

1. Interest rates can be *least likely* interpreted as:
 - A. discount rates.
 - B. opportunity costs.
 - C. sunk costs.
2. The following information is provided regarding a security whose nominal interest rate is 10%:
 - The real risk-free rate of return is 4%
 - The default risk premium is 1%
 - The maturity risk premium is 1%
 - The liquidity risk premium is 2%

An investor wants to determine the inflation premium in the security's return. The inflation premium is *closest* to:

- A. 2%.
 - B. 4%.
 - C. 6%.
3. Which of the following fixed income instruments has the *highest* effective annual rate (EAR)?

	Compounding frequency	Annual interest rate
Instrument 1	Monthly	6.20%
Instrument 2	Quarterly	6.25%
Instrument 3	Continuously	6.00%

- A. Instrument 1.
 - B. Instrument 2.
 - C. Instrument 3.
4. How much must be invested today at 8% interest compounded quarterly to accumulate \$10,000 five years from today. The amount that must be invested today is *closest* to:
 - A. \$6,210.
 - B. \$6,730.
 - C. \$6,840.
5. The amount an investor will have in 10 years, if \$1000 is invested today at a continuously compounded rate of 7%, will be *closest* to:
 - A. \$2,014.
 - B. \$2,038.
 - C. \$2,044.

6. Nancy Scott is buying a house. She expects her budget to allow a monthly payment of \$2000 on a 20-year mortgage with a stated annual interest rate of 6 percent. If Ms. Scott puts a 15 percent down payment, the most she can pay for the house is *closest* to:
 - A. \$279,160.
 - B. \$328,425.
 - C. \$336,160.
7. A tenant pays rent of \$800 monthly due on the first day of every month. If the annual interest rate is 7 percent, the present value of a full year's rent is *closest* to:
 - A. \$9,245.
 - B. \$9,300.
 - C. \$9,355.
8. The preferred shares of Crane Industries are expected to pay a \$10 dividend forever, starting from the end of next year. If the required rate of return on equivalent investments is 9%. A share of Crane Industries preferred stock should be worth:
 - A. \$90.5.
 - B. \$111.1.
 - C. \$124.6.
9. An investment is expected to produce the cash flows of \$100, \$200 and \$300 at the end of the next three years. If the required rate of return is 10%, the present value of this investment is closest to:
 - A. \$456.65.
 - B. \$475.83.
 - C. \$481.59.
10. James Miller wants to save for his son's college tuition. He will have to pay \$40,000 at the end of each year for the four years that his son attends college. He has 6 years until his son starts college to save up for his tuition. Using a 8% interest rate compounded annually, the amount Miller would have to save each year for 6 years is *closest* to:
 - A. \$16,190.
 - B. \$18,060.
 - C. \$19,530.

Solutions

1. C is correct. Interest rates can be interpreted as
 - Required rate of return: It is the minimum rate that the investors require to make investments.
 - Discount rate: Future values can be discounted by interest rates to arrive at present values.
 - Opportunity costs: If instead of investing, you spend the money on something else, then you have given up the opportunity to earn interest.

Sunk cost is a cost that has already been incurred and cannot be recovered.

2. A is correct. Nominal interest rate = real risk-free rate of return + inflation premium + risk premiums (default, liquidity, maturity premiums)

Therefore,

$$\text{Inflation premium} = 10\% - 4\% - 1\% - 1\% - 2\% = 2\%$$

3. A is correct. Use the EAR (effective annual rate) to compare the investments:

Instrument	Formula	EAR
Instrument 1	$(1 + .062/12)^{12} - 1$	6.379%
Instrument 2	$(1 + .0625/4)^4 - 1$	6.398%
Instrument 3	$e^{(0.060 \times 1)} - 1$	6.183%

4. B is correct. Enter into your financial calculator $FV=10,000$, $N=5 \times 4 = 20$, $I/Y = 8/4 = 2$, $PMT=0$, and solve for PV. $PV = -6,729.71$
5. A is correct. The future value of an amount calculated using continuous compounding is:

$$FV = Pv \times e^{rt}$$
 Thus:

$$FV = 1000 \times e^{0.07 \times 10} = \$2,013.75$$
6. B is correct. The consumer's budget will support a monthly payment of \$2,000. Given a 20-year mortgage at 6 percent, the loan amount will be \$279,161 ($N = 20 \times 12 = 240$, $\%I = 6/12 = 0.5$, $PMT = 2,000$, $FV = 0$ solve for PV). If she makes a 15% down payment, then the most she can pay for the new house = $\$279,161 / (1 - 0.15) = \$328,424.7$.
7. C is correct. Using a financial calculator: $PMT = 800$, $I = 7/12 = 0.583$, $n=12$ Compute annuity due PV, $PV = \$9299.6$ (Put the calculator in BGN mode for annuity due calculations)
8. B is correct.

$$PV_{perpetuity} = \frac{PMT}{I/Y} = \frac{\$10}{0.09} = \$111.11$$

9. C is correct. Using your cash flow keys, (CF0=0), CF1 = 100, CF2 = 200, CF3= 300, I= 10
Compute PV, PV = \$481.59
10. B is correct. Using a financial calculator, we first need to calculate the total value of the tuition fees needed at the end of 6 years. Note that the first payment of 40,000 needs to be made 7 years from today.
N = 4, I/Y = 8, PMT = 40,000, FV = 0
Compute PV: PV= \$132,485.07. This is the amount of money needed at the end of 6 years.
- Using \$132,485.07 as the FV for the saving phase annuity, we compute yearly deposits with the inputs:
N=6, I/Y = 8, PV = 0, FV = 132,485.07
Compute PMT: PMT = \$18059.75 ~ \$18060.

R07 Discounted Cash Flow Applications

1. Introduction

This reading covers:

- Decision tools: net present value (NPV) and internal rate of return (IRR).
- Decision rules for NPV and IRR.
- Conflicting situations under IRR, and what rule to use in such cases.
- Calculating returns of a portfolio: money-weighted return and time-weighted return.
- Calculating different yield measures for money market instruments.

2. Net Present Value and Internal Rate of Return

Net present value and internal rate of return are often applied in all fields of finance. But perhaps the most common and well known application of these concepts is in the field of capital budgeting, where companies need to make decisions on which projects to pursue and which projects to reject.

2.1 Net Present Value and the Net Present Value Rule

The net present value (NPV) of an investment is the present value of its cash inflows minus the present value of its cash outflows. The steps to calculate NPV are as follows:

- Identify all the cash flows associated with an investment (both inflows and outflows).
- Determine the appropriate discount rate 'r' for the investment project. 'r' is also known as the cost of capital or the opportunity cost.
- Using the discount rate 'r', find the present value of each cash flow.
- Inflows are positive and outflows are negative. Sum the present values of all cash flows, including the initial investment. This value represents the investment's net present value.

Formula for computing NPV is:

NPV = present value of cash inflows – present value of cash outflows

$$NPV = \sum_{t=0}^N \frac{CF_t}{(1+r)^t}$$

where:

CF_t = the expected net cash flow at time t

N = the investment's projected life

r = the discount rate or opportunity cost

To understand the NPV concept, let us consider a simple example.

Example

A project requires an initial outlay of \$750,000. It is expected to produce \$200,000 in the first year, \$300,000 in the second year, and \$400,000 in the third year. The cost of capital for

this project is 10%. What is the NPV? Should the project be accepted?

Solution:

Formula Method:

Using the formula for NPV,

$$\begin{aligned} \text{NPV} &= \text{CF}_0 + \left[\frac{\text{CF}_1}{(1+r)^1} \right] + \left[\frac{\text{CF}_2}{(1+r)^2} \right] + \left[\frac{\text{CF}_3}{(1+r)^3} \right] \\ \text{NPV} &= -750,000 + \frac{200,000}{(1.1)^1} + \frac{300,000}{(1.1)^2} + \frac{400,000}{(1.1)^3} \\ \text{NPV} &= -19,722 \end{aligned}$$

While this method is conceptually simple, it can be tedious if the number of cash flows is large. It is much easier to use the financial calculator. The steps are outlined below:

Calculator Method:

Keystrokes	Explanation	Display
[2nd] [QUIT]	Return to standard mode	0
[CF] [2nd] [CLR WRK]	Clear CF Register	CF = 0
750 [+/-] [ENTER]	Initial Outlay (in 000's)	CF0 = -750
[↓] 200 [ENTER]	Enter CF at t = 1	C01 = 200
[↓] [↓] 300 [ENTER]	Enter CF at t = 2	C02 = 300
[↓] [↓] 400 [ENTER]	Enter CF at t = 3	C03 = 400
[↓] [NPV] [10] [ENTER]	Enter discount rate	I = 10
[↓] [CPT]	Compute NPV	-19.722

The present value of cash inflows is \$730,278 which is less than the cost of \$750,000. Value is being destroyed. The NPV is negative and hence this project should be rejected.

NPV Decision Rule

For independent projects (where you can make a decision on each project independently):

- If the NPV is positive → Accept the project.
- If the NPV is negative → Reject the project.

For mutually exclusive projects (where only one project can be accepted):

- Accept the project with the higher NPV as long as the NPV is positive.

2.2 The Internal Rate of Return and the Internal Rate of Return Rule

The internal rate of return (IRR) is the discount rate that makes the net present value equal to zero. It is 'internal' because it depends only on the cash flows of the investment; no external data is needed. The formula for IRR is as follows:

$$\text{NPV} = \text{CF}_0 + \left[\frac{\text{CF}_1}{(1 + \text{IRR})^1} \right] + \left[\frac{\text{CF}_2}{(1 + \text{IRR})^2} \right] + \dots + \left[\frac{\text{CF}_N}{(1 + \text{IRR})^N} \right] = 0$$

where:

CF_0 = usually the initial investment which is a cash outflow

CF_t = the expected net cash flow at time t

NPV = net present value of the investment

IRR = internal rate of return

The IRR is a single number which represents the return generated by a project. Consider a very simple project where the initial investment is \$100. One year later the amount received from this project is \$110. There are no other cash flows. If we apply the formula for IRR we get:

$$-100 + \frac{110}{1 + \text{IRR}} = 0$$

Solving this equation shows that $\text{IRR} = 0.1$ or 10%.

Example

Consider a project that requires an initial investment of \$150,000. Estimated cash flows for the following three years are \$50,000, \$100,000 and \$40,000 respectively. What is the IRR?

Solution:

We can set up an equation with the initial outflow equal to the present value of future cash flows and solve for the IRR:

$$CF_0 = \left[\frac{CF_1}{(1+\text{IRR})^1} \right] + \left[\frac{CF_2}{(1+\text{IRR})^2} \right] + \left[\frac{CF_3}{(1+\text{IRR})^3} \right]$$

Plugging in the values, we get:

$$\$150,000 = \left[\frac{\$50,000}{(1+\text{IRR})^1} \right] + \left[\frac{\$100,000}{(1+\text{IRR})^2} \right] + \left[\frac{\$40,000}{(1+\text{IRR})^3} \right]$$

While it is theoretically possible to solve the above equation, it is much simpler to use the financial calculator.

Keystrokes	Explanation	Display
[2nd] [QUIT]	Return to standard mode	0
[CF] [2nd] [CLR WRK]	Clear CF Register	CF = 0
150 [+/-] [ENTER]	Initial outlay (in 000's)	CF0 = -150
[↓] 50 [ENTER]	Enter CF at $t = 1$	C01 = 50
[↓] [↓] 100 [ENTER]	Enter CF at $t = 2$	C02 = 100
[↓] [↓] 40 [ENTER]	Enter CF at $t = 3$	C03 = 40
[↓] [IRR] [CPT]	Compute IRR	13.11%

The IRR Rule

For independent projects:

- If $\text{IRR} > \text{Opportunity cost} \rightarrow$ Accept the project.
- If $\text{IRR} < \text{Opportunity cost} \rightarrow$ Reject the project.

For mutually exclusive projects:

- Accept the project with the higher IRR, as long as the $\text{IRR} > \text{Opportunity cost}$.

IRR uses the opportunity cost of capital as the hurdle rate. For example, if at a given company the cost of capital (opportunity cost) is 10%, then only projects with $IRR > 10\%$ will be accepted.

Example

Bill is interested in a project which requires an investment of \$1.5 million. The project shall pay \$200,000 per year in perpetuity. The first cash flow will be received 1 year from today. The cost of capital is 8%. What is the IRR? Should Bill invest in this project?

Solution:

IRR can also be thought of as the discount rate which makes the initial outlay equal to the present value of future cash flows. Here the initial outlay is \$1,500,000. Because the project's cash flows are a perpetuity, the present value of future cash flows can be written as: $\$200,000/IRR$. Hence the equation becomes:

$$1,500,000 = \frac{200,000}{IRR}$$

We can solve for IRR as:

$$IRR = \frac{200,000}{\$1,500,000} = 0.133 \text{ or } 13.3\%$$

Since the IRR of 13.3% is greater than the cost of capital of 8%, Bill should invest in this project.

2.3 Problems with the IRR Rule

NPV and IRR rules give the same accept or reject decision when projects are independent. However, when dealing with mutually exclusive projects, it is possible that the highest NPV project is not the same as the highest IRR project. This is called a ranking conflict.

IRR and NPV can rank projects differently when:

- The size or scale of the projects differs.
- Timing of the projects' cash flows differs.

To illustrate the first point (difference in size), consider a company with \$100 million available to invest. It has two investment projects as shown below:

Project	Investment at $t = 0$	Cash flow at $t = 1$	IRR (%)	NPV at 10%
A	-100	120	20%	10
D	-1,000	1,150	15%	45

The IRR rule ranks Project A first because of the higher IRR. The NPV rule, however, ranks Project D first because of the higher NPV.

To illustrate the second point (difference in cash flow timing), consider Projects A and C as shown in the table below:

Project	CF0	CF1	CF2	CF3	IRR (%)	NPV at 10%
---------	-----	-----	-----	-----	---------	------------

A	-100	120	0	0	20	10
C	-100	0	0	170	19	28

The IRR rule ranks Project A first because of the higher IRR. The NPV rule, however, ranks Project C first because of the higher NPV.

Whenever there is a conflict in ranking between the IRR rule and the NPV rule, the NPV rule should be used to decide between mutually exclusive projects. This is because the NPV represents the expected addition to shareholder wealth from an investment. The maximization of shareholder wealth is a basic financial objective of a company and hence, the NPV rule must be given preference.

Therefore, in the first case you should select Project D over Project A. Similarly, in the second case you select Project C over Project A.

3. Portfolio Return Measurement

Portfolio performance measurement involves calculating returns in a logical and consistent manner. For instance, determining the returns of a portfolio helps in comparing and ranking different mutual funds. In this section, we cover the following return measures and identify situations when these measures are to be used:

- Holding period return
- Money-weighted rate of return (MWRR)
- Time-weighted rate of return (TWRR)

Holding Period Return

Holding period return (HPR) is the return that an investor earns over a specified holding period. The holding period can range from days to years. The formula for calculating HPR for an investment that makes one-time cash payment at the end of the holding period is given below:

$$\text{HPR} = \frac{P_1 - P_0 + D_1}{P_0} = \frac{\text{ending value} - \text{beginning value} + \text{cash flow}}{\text{beginning value}}$$

where:

P_0 = initial investment

P_1 = price received at the end of the holding period

D_1 = cash paid by the investment at the end of the holding period

Example

Assume we buy a stock for \$50. Six months later, the stock price goes up to \$53 and we receive a dividend of \$2. Calculate the holding period return.

Solution:

The return for the six-month holding period is given below:

$$\text{HPR} = \frac{53 + 2 - 50}{50} = 0.10 = 10\%$$

3.1 Money-Weighted Rate of Return

The money-weighted rate of return is the internal rate of return (IRR) of a project or an investment.

Let us consider a simple example to illustrate this point. At time $t = 0$, an investor buys a share for \$20.00. At the end of the Year 1, he receives a dividend of \$0.50 and purchases another share for \$22.50. At the end of the Year 2, he sells both shares for \$23.50 each after receiving another dividend of \$0.50 per share. What is the money-weighted return?

Since the money-weighted return is the IRR, we can use a financial calculator. The first step is to determine the net cash flows for every period. This is illustrated in the table below:

<i>Time (end of period)</i>	<i>Outflow (-)</i>	<i>Inflow (+)</i>	<i>Net cash flow</i>
0	-\$20.00 To purchase the first share		-\$20.00
1	-\$22.50 To purchase the second share	\$0.50 Dividend received on first share	-\$22.00
2		Dividend received = \$0.50 x 2 shares = \$1.00 From sales of 2 shares = \$47	+48.00

After entering these cash flows, use the calculator's IRR function to find the money-weighted rate of return as 9.39%.

3.2 Time-weighted Rate of Return

The time-weighted rate of return measures the compound growth rate of \$1 initially invested in the portfolio over a stated measurement period. The time-weighted return can be calculated using the following steps:

1. Break the overall evaluation period into sub-periods based on the dates of cash inflows and outflows.
2. Calculate the holding period return on the portfolio for each sub-period.
3. Link or compound holding period returns to obtain an annual rate of return for the year (the time-weighted rate of return for the year).
4. If the investment is for more than a year, take the geometric mean of the annual returns to obtain the time-weighted rate of return over that measurement period.

Consider the same example we discussed above with the following cash flows:

<i>Time</i>	<i>Outflow</i>	<i>Inflow</i>
0	\$20.00 to purchase the first share	

1	\$22.50 to purchase the second share	\$0.50 dividend received on first share
		\$1.00 dividends (\$0.50 x 2 shares); \$47.00 from selling 2 shares @ \$23.50 per share

Calculating the TWRR for this example is relatively simple because cash flows only occur at the start/end of every year. We will follow the steps mentioned earlier:

Steps 1: Break into evaluation period and value the portfolio at start/end of every period.

- Value of the portfolio at the start of Year 1 ($t = 0$) is \$20.00.
- Value of portfolio at the end of Year 1 ($t = 1$) before the purchase of the new share is $22.50 + 0.50 = \$23.00$. Note that the dividend of \$0.50 on the first share is received at the end of Year 1.
- Value of the portfolio at the start of Year 2 ($t = 1$) after the purchase of the second share is $22.50 + 22.50 = \$45.00$. The dividend of \$0.50 from the first share is paid out and is not considered as part of the portfolio.
- Value of the portfolio at the end of Year 2 ($t = 2$) is $23.50 + 23.50 + 0.50 + 0.50 = \48.00 . Both shares pay a dividend of \$0.50 at the end of the second year.

Step 2: Calculate the holding period return on the portfolio for each sub-period.

- In this question the cash flows are taking place at the start/end of each period. Hence there are no sub-periods. *Scenarios involving sub-periods will be covered in the next example.*

Step 3: Link or compound holding period returns to obtain an annual rate of return for the year.

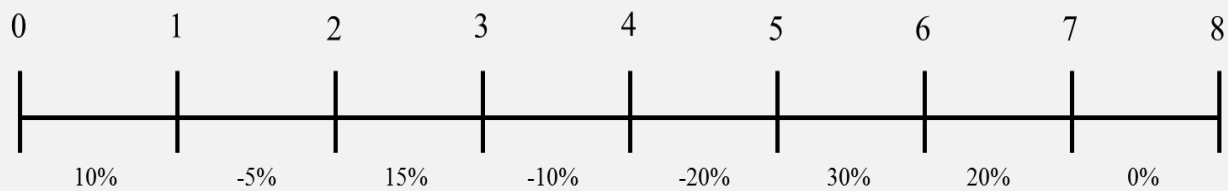
- The annual rate of return is based on the portfolio value at the start and end of each period.
- The portfolio value at the start of Year 1 was \$20.00 and the value at the end of Year 1 was \$23.00. Hence the holding period return was 15.00%.
- The portfolio value at the start of Year 2 was \$45.00 and the value at the end of Year 2 was \$48.00. Hence the holding period return was 6.67%.

Step 4: If the investment is for more than a year, take the geometric mean of the annual returns to obtain the time-weighted rate of return over that measurement period.

The TWRR is calculated as: $(1.15 * 1.067)^{\frac{1}{2}} - 1 = 0.1077 = 10.77\%$.

Example

Consider an investment where cash flows occur at the start/end of every quarter. Here each quarter is considered a sub-period. The return for each sub-period has already been calculated and is shown below:



Calculate the time-weighted return.

Solution:

Step 1 (break evaluation into sub-periods) and step 2 (calculate HPR for sub-periods) has been done for you.

Step 3: Link the quarterly returns to determine the return for Years 1 and 2 respectively.

For year 1: $(1 + 0.10) (1 - 0.05) (1 + 0.15) (1 - 0.10) = 1.0816$

For year 2: $(1 - 0.20) (1 + 0.30) (1 + 0.20) (1 + 0) = 1.2480$

Step 4: Determine the annualized return by taking the geometric mean.

$TWRR = (1.0816 \times 1.2480)^{1/2} - 1 = 0.1618 = 16.18\%$.

Money weighted v/s time weighted returns

- The money-weighted rate of return is impacted by the timing and amount of cash flows.
- The time-weighted rate of return is not impacted by the timing and amount of cash flows.
- The time-weighted return is an appropriate performance measure if the portfolio manager does not control the timing and amount of investment.
- On the other hand, money-weighted return is an appropriate measure if the portfolio manager has control over the timing and amount of investment.

4. Money Market Yields

The money market is the market for short-term debt instruments (one year maturity or less). Some instruments require the issuer to repay the lender the amount borrowed plus the interest. Others are pure discount instruments that pay interest as the difference between the amount borrowed and the amount paid back.

The following yield measures are used when dealing with money market instruments:

- Bank discount yield
- Holding period yield
- Effective annual yield
- Money market yield
- Bond equivalent yield

Bank Discount Yield

The bank discount yield is computed as follows:

$$BDY = \frac{D}{F} \times \frac{360}{t}$$

where:

BDY = the annualized yield on a bank discount basis

D = the dollar discount, which is equal to the difference between the face value of the bill and the purchase price

F = face value of the bill

t = the actual number of days until maturity

Example

Consider a T-bill with a face value (or par value) of \$100,000 and 150 days until maturity which is selling for \$98,000. The dollar discount, D, is \$2,000. Calculate the bank discount yield.

Solution:

$$BDY = \frac{2,000}{100,000} \times \frac{360}{150} = 0.048 = 4.8\%$$

The bank discount yield (BDY) is the industry standard measure for quoting T-bills. However, the BDY is not an accurate measure of investors' returns for the following three reasons:

1. The yield is based on the face value of the bond, not on its purchase price. In the above example, the dollar discount of \$2,000 is divided by the face value of \$100,000. A more accurate measure of return can be obtained if we divide by the investment amount (or the T-bill price) of \$98,000.
2. The yield is annualized based on a 360-day year rather than a 365-day year.
3. The bank discount yield annualizes with simple interest. In the above example the 150-day return is multiplied by 360/150. This ignores the opportunity to earn interest on interest (compound interest).

Holding Period Return

It is the return an investor will earn by holding the instrument to maturity. It is also known as holding period yield (HPY).

$$HPY = \frac{P_1 - P_0 + D_1}{P_0}$$

where:

P_0 = initial purchase price of the instrument

P_1 = price received for the instrument at maturity

D_1 = cash paid by the instrument at maturity

Note: This concept has been discussed earlier in Section 3.

Example

Consider a T-bill with a face value of \$100,000 and 150 days until maturity which is selling for \$98,000. Determine the holding period yield.

Solution:

$$\text{HPY} = (100,000 - 98,000 + 0) / 98,000 = 0.0204 = 2.04\%$$

Money Market Yield

The money market yield is computed by annualizing the HPY assuming simple interest and a 360-day year. The formula is as follows:

$$\text{MMY} = \frac{\text{HPY} \times 360}{t}$$

where:

HPY = holding price yield

360 = number of days in a year

t = the actual number of days until maturity

Example

Consider a T-bill with a face value (or par value) of \$100,000 and 150 days until maturity which is selling for \$98,000. Calculate the money market yield.

Solution:

We earlier calculated the holding period yield as 2.04%.

$$\text{MMY} = \frac{0.0204 \times 360}{150} = 0.0490 = 4.90\%$$

Effective Annual Yield

The most accurate measure of investors' returns is the effective annual yield (EAY). The effective annual yield is calculated as follows:

$$\text{EAY} = (1 + \text{HPY})^{\frac{365}{t}} - 1$$

where:

HPY = holding price yield

365 = number of days in a year

t = the actual number of days remaining to maturity

The effective annual yield addresses the three weakness of the bank discount yield:

1. The HPY correctly gives a return with respect to the investment amount and not the face value.
2. The HPR is annualized based on a 365-day year, not a 360-day year.
3. By using an exponent, compound interest is being used rather than simple interest.

Example

Consider a T-bill with a face value (or par value) of \$100,000 and 150 days until maturity which is selling for \$98,000. Determine the effective annual yield.

Solution:

In an earlier example, we calculated the holding period yield as 2.04%.

$$\text{EAY} = (1 + 0.0204)^{\frac{365}{150}} - 1 = 0.0504 = 5.04\%$$

Bond Equivalent Yield

Bond equivalent yield is computed as:

$$\text{Bond equivalent yield} = 2 \times \text{semiannual yield}$$

Most bonds make coupon payments every six months. Consider a bond worth \$100 and which makes a coupon payment of \$4 semi-annually. The semi-annual or six-month yield on this bond is $4/100 = 4\%$. The bond equivalent yield is two times the six-month yield: $2 \times 4\% = 8\%$. Note that for this bond the effective annual yield will be $1.04^2 - 1 = 0.0816$ or 8.16%.

Instructor's Note:

Here is a suggestion on how to remember the various money market yields.

Bank discount yield (BDY) = $D/F \times 360/t$. Remember from the name that there is a discount involved. The discount is relative to the face value. Hence we have D/F in the BDY expression. Next D/F needs to be annualized using simple interest so we multiply by $360/t$ where t is the number of days to maturity. So the overall formula becomes: $\text{BDY} = D/F \times 360/t$. Also remember the three limitations of this measure: 1) the discount is divided by face value rather than price, 2) the use of 360 days in a year rather than 365 days and 3) the use of simple interest to annualize rather than compound interest.

Holding period yield (HPY) = $(\text{Increase in price} + \text{any other cash flow}) / \text{Investment amount}$. As the name implies this is the yield or return for a given holding period. It is defined as the gain (increase in price plus any dividend or coupon payment) over the holding period divided by the original investment amount. By definition this yield measure is not annualized.

Money market yield (MMY) = $\text{HPY} \times 360/t$. Here we are annualizing the HPY using a 360-day year and simple interest. Since the HPY is based on the gain divided by the investment amount, MMY addresses the first and most significant limitation of the BDY. The other two limitations remain.

Effective annual yield (EAY) = $(1 + \text{HPY})^{365/t} - 1$. This measure compounds the HPY using a 365-day year. Hence it addresses all three limitations of the BDY.

Converting between yield measures

The table below summarizes how to convert between different yield measures:

The curriculum outlines a formula for converting a bank discount yield to a money market

yield, but it is easier and more intuitive to use the process shown in the table below:

<i>Conversion</i>	<i>Formula / Method</i>
HPY to MMY	$MMY = \frac{HPY \times 360}{t}$
HPY to EAY	$EAY = (1 + HPY)^{\frac{365}{t}} - 1$
HPY to BDY (bank discount yield)	Given the face value and HPY, compute price and the discount. Using the discount, face value and number of days to maturity, compute the bank discount yield.
BDY to EAY	From the BDY, compute the discount. From the discount, compute the HPY. From the HPY, compute the EAY.
BDY to MMY	From the BDY, compute the discount and the price. From the discount and the price, compute the HPY. From the HPY, compute the MMY.

Example

Consider a T-bill with a face value of \$100,000 and 150 days until maturity. Assume the bank discount yield is given as 4.80%. Determine the holding period yield and the money market yield.

Solution:

1. Compute the discount: $0.048 \times \$100,000 \times \frac{150}{360} = \$2,000$ and the price:
 $\$100,000 - \$2,000 = \$98,000$
2. From the discount and price, we can compute the HPY:
 $\frac{\$2,000}{\$98,000} = 0.0204$
3. From the HPY, compute the MMY: $0.0204 \times \frac{360}{150} = 0.04896 = 4.90\%$

Summary

LO.a: Calculate and interpret the net present value (NPV) and the internal rate of return (IRR) of an investment.

The net present value (NPV) of an investment is the present value of its cash inflows minus the present value of its cash outflows.

The internal rate of return (IRR) is the discount rate that makes the net present value equal to zero.

LO.b: Contrast the NPV rule to the IRR rule, and identify problems associated with the IRR rule.

The NPV rule for independent projects is to accept the project if the investment's NPV is positive and reject the project if NPV is negative. However, for mutually exclusive projects the investor must choose the project with the highest positive NPV.

The IRR rule for independent projects is to accept the project if the investment's IRR is greater than the cost of capital and reject the project if it is less than the cost of capital. In case of mutually exclusive projects, the project with the highest IRR must be selected.

There is a theoretical limitation of IRR, whereby interim cash flows are assumed to be reinvested at the IRR rate and not at the cost of capital. This leads to a ranking conflict, i.e. when deciding between projects which are not independent, the NPV and IRR rules do not always lead to the same decision. Whenever there is a conflict in ranking between the IRR rule and the NPV rule, the NPV rule should be used to decide between mutually exclusive projects.

LO.c: Calculate and interpret a holding period return (total return).

Holding Period Return (HPR) is the return that an investor earns over a specified holding period.

$$\text{HPR} = \frac{P_1 - P_0 + D_1}{P_0}$$

where: P_0 = initial investment, P_1 = price received at the end of the holding period, D_1 = cash paid by the investment at the end of the holding period.

LO.d: Calculate and compare the money-weighted and time-weighted rates of return of a portfolio and evaluate the performance of portfolios based on these measures.

The money-weighted rate of return accounts for the timing and amount of all cash flows into and out of a portfolio. It is simply the internal rate of return.

The time-weighted rate of return measures the compound rate of growth of \$1 initially invested in the portfolio over a stated measurement period.

Money weighted v/s time weighted returns

- The money-weighted rate of return is impacted by the timing and amount of cash flows.
- The time-weighted rate of return is not impacted by the timing and amount of cash flows.
- The time-weighted return is an appropriate performance measure if the portfolio manager does not control the timing and amount of investment.
- On the other hand, money-weighted return is an appropriate measure if the portfolio manager has control over the timing and amount of investment.

LO.e: Calculate and interpret the bank discount yield, holding period yield, effective annual yield, and money market yield for US Treasury bills and other money market instruments.

$$\text{BDY} = \left(\frac{D}{F} \right) * \left(\frac{360}{t} \right)$$

$$\text{HPY} = \frac{P_1 - P_0 + D_1}{P_0}$$

$$\text{EAY} = (1 + \text{HPY})^{365/t} - 1$$

$$\text{MMY} = \frac{\text{HPY} * 360}{t}$$

LO.f: Convert among holding period yields, money market yields, effective annual yields, and bond equivalent yields.

HPY to BDY (bank discount yield)	Given the face value and HPY, compute price and the discount. Using the discount, face value and number of days to maturity, compute the bank discount yield.
BDY to EAY	From the BDY, compute the discount. From the discount, compute the HPY. From the HPY, compute the EAY.
BDY to MMY	From the BDY, compute the discount and the price. From the discount and the price, compute the HPY. From the HPY, compute the MMY.

Bond-equivalent yield = 2 * semi-annual YTM

Practice Questions

1. The incremental after-tax cash flows of a project are given below:

Year	0	1	2	3	4
Cash flow (€)	-100,000	50,000	40,000	20,000	6,000

The discount rate for evaluating the project is taken as 10%. The NPV (in €) of the project is *closest* to:

- A. -2,363.
B. 1,586.
C. 2,458.
2. The Indian government wishes to invest in a project that shall require an initial investment of \$20 million, and will produce positive cash flows of \$5 million for the first three years, and \$4 million for the next two years. What is the approximate internal rate of return (IRR) for the investment?
- A. 3%.
B. 4%.
C. 5%.
3. As a project manager, Ronald Parker has to choose between three mutually exclusive projects: A, B and C. He has the following information from his staff regarding the three projects:

	NPV	IRR
A	\$11,000	8%
B	\$15,000	10%
C	\$10,000	12%

Based on the information given, which project would be *most* appropriate for Parker's department?

- A. Project A.
B. Project B.
C. Project C.
4. Nancy Miller purchased 100 shares of a company at a price \$15 per share on 1 January. She sold all the stocks on 30 June of the same year at price \$ 18 per share. She also received dividends totaling \$80 on 30 June. The holding period return on the investment is *closest* to:
- A. 20%.
B. 25%.
C. 30%.
5. An investor purchases one share of a company for \$50. Exactly one year later, the

company pays a dividend of \$5.00 per share. This is followed by two more annual dividends of \$6.00 and \$6.50 in successive years. Upon receiving the third dividend, the investor sells the share for \$60. The money-weighted rate of return on this investment is *closest* to:

- A. 15%.
- B. 16%.
- C. 17%.

6. An investor purchases 100 shares of a company. The table below outlines the history of his investment:

Time	Activity	Price per Share	Dividend per Share
Begn of Year 1	Buy 100 shares	\$10	
End of Year 1	Buy 10 shares	\$11	\$1
End of Year 2		\$13	\$1.5
End of Year 3	Sell 110 shares	\$12	

Assuming that the investor does not reinvest his dividends, which are tax-free, the time-weighted rate of return on the investment is *closest* to:

- A. 12.2%.
 - B. 13.7%.
 - C. 14.8%.
7. A T-Bill with a par value of \$1,000 and 120 days to maturity is trading at a price of \$980. The bank discount yield of the T-Bill is *closest* to:
- A. 5%
 - B. 6%
 - C. 7%.
8. A 180-day U.S. Treasury bill with a face value of \$1,000 sells for \$990 when issued. Assuming an investor holds the bill to maturity, the investor's money market yield is *closest* to:
- A. 1.19%.
 - B. 2.02%.
 - C. 3.50%.
9. A Treasury bill offers a bank discount yield of 5% and has 180 days to maturity. The effective annual yield on the instrument is *closest* to:
- A. 4.88%.
 - B. 5.26%.
 - C. 5.79%.

Solutions

1. A is correct.

NPV of a project = Present value of the expected cash inflows – Present value of the expected cash outflows.

Enter the given cash flows and the given discount rate into a financial calculator and solve for NPV. $CF_0 = -100,000$, $CF_1 = 50,000$, $CF_2 = 40,000$, $CF_3 = 20,000$, $CF_4 = 6,000$, $i = 10\%$. Compute PV. The NPV is $-2,363$.

2. C is correct. IRR is defined as the rate of return that equates the PV of the cash inflows to the PV of the cash outflows.

C is correct. Enter the given cash flows in the financial calculator

$CF_0 = -20$ million

$CF_1 = 5$ million

$CF_2 = 5$ million

$CF_3 = 5$ million

$CF_4 = 4$ million

$CF_5 = 4$ million

IRR Compute = 5.07%

3. B is correct. B is correct. Project B has the highest NPV among the three projects and thus results in the greatest addition to shareholder wealth. While there is a conflict among the NPV and IRR rules for projects B and C, NPV rule is to be given preference for its superiority over IRR and hence B would be the most appropriate choice.

4. B is correct.

$$HPR = \frac{(P_1 - P_0 + D_1)}{P_0}$$

$$(18 - 15 + 80/100) / 15 = 0.2533 = 25.33\%.$$

5. C is correct. The money-weighted rate of return is the internal rate of return (IRR) of the cash flows associated with the investment. Use the cash flow (CF) function of a financial calculator and enter $CF_0 = -50$, $CF_1 = 5$, $CF_2 = 6$, and $CF_3 = 66.5$. Calculate the IRR. The answer is 17.15%

6. B is correct. $TWR = \sqrt[3]{[(11 + 1)/10] \times [(13 + 1.5)/11] \times [(12/13)]} - 1 = 0.1344 = 13.45\%$.

7. B is correct. The bank discount yield = $\text{Discount} / \text{Face Value} \times 360 / \text{No of days to maturity} = 20/1000 \times 360/120 = 0.06 = 6\%$.

8. B is correct. The money market yield is: $2.02\% = [(1,000/990) - 1] \times (360/180)$.

$$MMY = HPY \times \frac{360}{t}$$

9. B is correct. First, calculate the initial price (P_0) of the T-bill:

$$0.05 = D/100 \times 360/180$$

$$D = 2.5$$

$$P_0 = 100 - 2.5 = 97.5$$

Then, calculate the holding period yield (HPY) (recall that T-bills are pure discount instruments and do not pay coupons):

$$HPY = (P_t - P_0) \div P_0$$

$$HPY = (100 - 97.5) \div 97.5 = 0.0256$$

Finally, convert the HPY into effective annual yield:

$$EAY = (1 + HPY)^{365/t} - 1$$

$$EAY = (1 + 0.0256)^{365/180} - 1 = 0.05259 = 5.26\%.$$

R08 Statistical Concepts and Market Return

1. Introduction

Statistics is a powerful tool for analyzing data and drawing conclusions. As investors, we are concerned about the returns on our investments and the distribution of those returns. We need this information to evaluate our investments.

For example, we often look for central tendency. Let's say that the stock market has returned on average 14% over the last 10 years. Obviously, we are concerned with this average number but we are also concerned with the dispersion which tells us how spread out the returns have been. One of the simplest measures of dispersion is range. Let's say that over the last 10 years the stock market has ranged between -20% and +35%. This tells us about the riskiness of our investments.

In this reading, we will study statistical methods that allow us to summarize return distributions.

Specifically, we will explore four properties of return distributions:

- Where the returns are centered (central tendency).
- How far returns are dispersed from their center (dispersion).
- Whether the distribution of returns is symmetrically shaped or lopsided (skewness).
- Whether extreme outcomes are likely (kurtosis).

2. Some Fundamental Concepts

2.1 The Nature of Statistics

The term statistics can have two broad meanings, one referring to data and the other to method used to analyze data. Statistical methods include:

- **Descriptive statistics:** It refers to how large data sets can be summarized effectively to describe their important characteristics.
- **Inferential statistics:** It refers to making forecasts, estimates or judgments about a large data set based on a small representative set.

The focus of this reading is on descriptive statistics. We will cover inferential statistics in a later reading.

2.2 Populations and Samples

A 'population' is defined as all members of a specified group. A 'parameter' describes the characteristics of a population.

A 'sample' is a subset drawn from a population. A 'sample statistic' describes the characteristic of a sample.

All stocks listed on the exchange of a country is an example of population. If 30 stocks are selected from among the listed stocks, then this is a sample.

2.3 Measurement Scales

The different types of measurement scales are:

- **Nominal scales:** These scales categorize data but do not rank them. Hence, they are often considered the weakest level of measurement. An example would be the classification of mutual funds according to the investment strategy followed – growth fund, value fund, income fund, emerging equity fund etc.
- **Ordinal scales:** These scales sort data into categories that are ordered with respect to some characteristic. An example is Standard & Poor's star ratings for mutual funds. One star represents the group of mutual funds with the worst performance. Similarly, groups with two, three, four and five stars represent groups with increasingly better performance.
- **Interval scales:** These scales not only rank data, but also ensure that the differences between scale values are equal. The Celsius and Fahrenheit scales are examples of such scales. The difference in temperature between 10°C and 11°C is the same amount as the difference between 40°C and 41°C. However, these scales do not have a true zero. For example, 0°C does not represent the absence of temperature. It is simply the freezing point of water; it is not a true zero.
- **Ratio scales:** These scales have all the characteristics of interval scales as well as a true zero point as the origin. This is the strongest level of measurement. The rate of return on an investment is measured on a ratio scale. A return of 0% means the absence of any return.

3. Summarizing Data Using Frequency Distributions

A frequency distribution is a tabular display of data summarized into a relatively small number of intervals. In order to construct a frequency distribution, we can follow the following procedure:

1. Define the intervals.
2. Tally the observations i.e. assign observations to intervals.
3. Count the observations in each interval.

Example

Say you are evaluating 100 stocks with prices ranging from 46 to 65 that are divided into the following four intervals of stock price each having a width of 5:

46-50, 51-55, 56-60 and 61-65. Assume the number of stocks whose prices fall in each of these intervals are 25, 35, 29, and 11 respectively.

Calculate the cumulative frequency, relative frequency, and the cumulative relative frequency for the stock prices given the set of intervals above.

Solution:

Stock Price	Absolute Frequency	Cumulative Frequency	Relative Frequency	Cumulative Relative Frequency
46-50	25	25	0.25	0.25
51-55	35	60	0.35	0.60
56-60	29	89	0.29	0.89
61-65	11	100	0.11	1.00

Absolute frequency: The actual number of observations in a given interval is called the absolute frequency, or simply the frequency, which is given here.

Cumulative frequency: For an interval, it is calculated as the sum of the absolute frequencies of all intervals lower than and including that interval.

Relative frequency: It is the absolute frequency of each interval divided by the total number of observations.

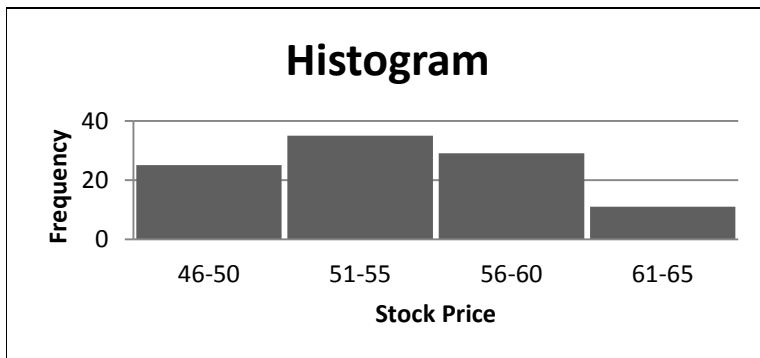
Cumulative relative frequency: For an interval, it is calculated as the sum of the relative frequencies of all intervals lower than and including that interval.

4. The Graphic Presentation of Data

4.1 The Histogram

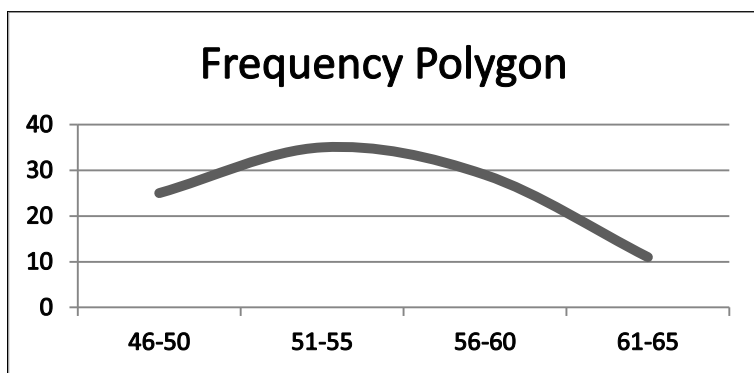
It is a bar chart of data that has been grouped together into a frequency distribution. The height of each bar is equal to the absolute frequency of each interval.

The advantage of the visual display is that we can quickly see where most of the observations lie.

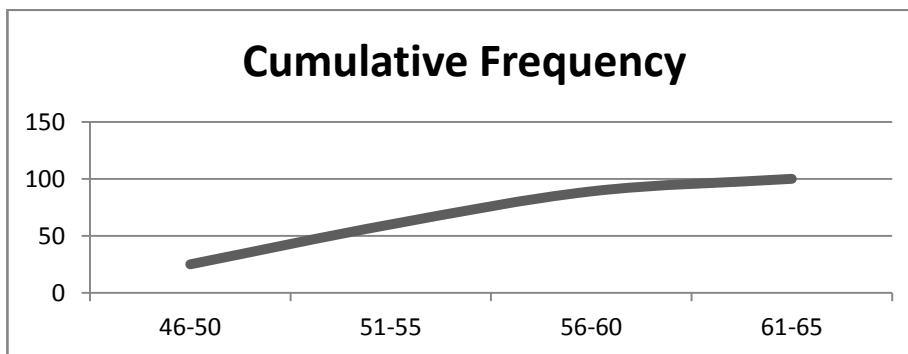


4.2 The Frequency Polygon and the Cumulative Frequency Distribution

A frequency polygon plots the midpoints of each interval on the X-axis and the absolute frequency of that interval on the Y-axis. Each point is then connected with a straight line.



Another graphical tool is the cumulative frequency distribution. Such a graph can plot either the cumulative frequency or cumulative relative frequency against the upper interval limit. The cumulative frequency distribution allows us to see how many or what percent of the observations lie below a certain value. The figure below is an example of a cumulative frequency distribution.



5. Measures of Central Tendency

5.1 The Arithmetic Mean

The arithmetic mean is the sum of the observations divided by the number of observations. It is the most frequently used measure of the middle or center of data.

The Population Mean

The population mean is the arithmetic mean computed for a population. It is expressed as:

$$\mu = \frac{\sum_{i=1}^N X_i}{N}$$

where:

N is the number of observations in the entire population and

X_i is the i^{th} observation

Consider the stock returns of a company over the last 10 years: 2%, 5%, 4%, 7%, 8%, 8%, 12%, 10%, 8%, and 5%.

For this data set, the population mean can be computed as:

$$\mu = \frac{2 + 5 + 4 + 7 + 8 + 8 + 12 + 10 + 8 + 5}{10} = 6.9\%$$

The Sample Mean

The sample mean is calculated like the population mean, except that we use the sample values. It is expressed as:

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n}$$

where: n is the number of observations in the sample.

If the sample data is: 8, 12, 10, 8 and 5, the sample mean can be calculated as:

$$\bar{X} = \frac{8 + 12 + 10 + 8 + 5}{5} = 8.6$$

5.2 The Median

The median is the midpoint of a data set that has been sorted into ascending or descending order.

For odd number of observations: 2,5,7,11,14 → Median = 7

For even number of observations: 3, 9, 10, 20 → Median = (9 + 10)/2 = 9.5

As compared to a mean, a median is less affected by extreme values (outliers).

5.3 The Mode

The mode is the most frequently occurring value in a distribution.

For the following data set: 2, 4, 5, 5, 7, 8, 8, 8, 10, 12 → Mode = 8

A distribution can have more than one mode, or even no mode. When a distribution has one mode it is said to be unimodal. If a distribution has two or three modes, it is called bimodal or trimodal respectively.

5.4 Other Concepts of Mean

The Weighted Mean

Here different observations are given different proportional influence on the mean. The formula for the weighted mean is:

$$\bar{X}_W = \sum_{i=1}^n w_i X_i$$

where: the sum of the weights equals 1; that is $\sum_{i=1}^n w_i = 1$

Example

Consider an investor with a portfolio of three stocks. \$40 is invested in A, \$60 in B and \$100 in C. If returns were 5% on A, 7% on B and 9% on C, compute the portfolio return using the

weighted mean.

Solution:

$$\left(\frac{40}{200}\right) \times 5\% + \left(\frac{60}{200}\right) \times 7\% + \left(\frac{100}{200}\right) \times 9\% = 7.6\%$$

An arithmetic mean is a special case of a weighted mean where all observations are equally weighted by the factor $1/n$.

The Geometric Mean

The most common application of the geometric mean is to calculate the average return of an investment. The formula is:

$$R_G = [(1 + R_1)(1 + R_2) \dots (1 + R_n)]^{\frac{1}{n}} - 1$$

Example

The return over the last four periods for a given stock is: 10%, 8%, -5% and 2%. Calculate the geometric mean.

Solution:

$$[(1 + 0.10)(1 + 0.08)(1 - 0.05)(1 + 0.02)]^{\frac{1}{4}} - 1 = 0.0358 = 3.58\%$$

Given the returns shown above, \$1 invested at the start of period 1 grew to:

$\$1.00 \times 1.10 \times 1.08 \times 0.95 \times 1.02 = \1.151 . If the investment had grown at 3.58% every period, \$1.00 invested at the start of period 1 would have increased to:

$\$1.00 \times 1.0358 \times 1.0358 \times 1.0358 \times 1.0358 = \1.151 . As expected, both scenarios give the same answer. 3.58% is simply the average growth rate per period.

Instructor's Note

In the reading on Discounted Cash Flow Applications, we used the geometric mean to calculate the time-weighted rate of return.

The Harmonic Mean

The harmonic mean is a special type of weighted mean in which an observation's weight is inversely proportional to its magnitude. The formula for a harmonic mean is:

$$X_H = \frac{n}{\sum_{i=1}^n \frac{1}{X_i}}$$

where: $X_i > 0$ for $i = 1, 2 \dots n$, and n is the number of observations

The harmonic mean is used to find average purchase price for equal periodic investments.

Example

An investor purchases \$1,000 of a security each month for three months. The share prices are \$10, \$15 and \$20 at the three purchase dates. Calculate the average purchase price per share for the security purchased.

Solution:

The average purchase price is simply the harmonic mean of \$10, \$15 and \$20.

The harmonic mean is: $\frac{3}{\frac{1}{\$10} + \frac{1}{\$15} + \frac{1}{\$20}} = \13.85 .

A more intuitive way of solving this is total money spent purchasing the shares divided by the total number of shares purchased.

Total money spent purchasing the shares = \$1,000 x 3 = \$3,000

Total shares purchased = sum of shares bought each month = $\frac{\$1,000}{10} + \frac{\$1,000}{15} + \frac{\$1,000}{20} = 100 + 66.67 + 50 = 216.67$

Average purchase price per share = $\frac{\$3,000}{216.67} = \13.85

Comparison of AM, GM and HM

- If the returns are constant over time: AM = GM = HM.
- If the returns are variable: AM > GM > HM.
- The greater the variability of returns over time, the more the arithmetic mean will exceed the geometric mean.

6. Other Measures of Location: Quantiles

A quantile is a value at or below which a stated fraction of the data lies. Some examples of quantiles include:

- **Quartiles:** The distribution is divided into quarters.
- **Quintiles:** The distribution is divided into fifths.
- **Deciles:** The distribution is divided into tenths.
- **Percentile:** The distribution is divided into hundredths.

The formula for the position of a percentile in a data set with n observations sorted in ascending order is:

$$L_y = \frac{(n + 1)y}{100}$$

where:

y is the percentage point at which we are dividing the distribution.

n is the number of observations.

L_y is the location (L) of the percentile (P_y) in an array sorted in ascending order.

Some important points to remember are:

- When the location, L_y , is a whole number, the location corresponds to an actual observation.
- When L_y is not a whole number or integer, L_y lies between the two closest integer numbers (one above and one below) and we use linear interpolation between those

two places to determine P_y .

Example

Consider the data set:

47 35 37 32 40 39 36 34 35 31 44

1. Find the 75th percentile point
2. Find the 1st quartile point
3. Find the 5th decile point
4. Find the 6th decile point.

Solution to 1:

First arrange the data in ascending order:

31, 32, 34, 35, 35, 36, 37, 39, 40, 44, 47

Location of the 75th percentile is the:

$$L_{75} = (11 + 1) (75/100) = 9^{\text{th}} \text{ value. i.e. } P_{75} = 40$$

With a small data set, such as this one, the location and the value is approximate. As the data set becomes larger, the location and percentile value estimates become more precise.

Solution to 2:

Location of the 1st quartile is the:

$$L_{25} = (11 + 1) (25/100) = 3^{\text{rd}} \text{ value. i.e. } P_{25} = 34$$

Solution to 3:

Location of the 5th decile is the:

$$L_{50} = (11 + 1) (50/100) = 6^{\text{th}} \text{ value. i.e. } P_{50} = 36$$

Solution to 4:

$$L_{60} = (11 + 1) (60/100) = 7.2$$

Use linear interpolation, which estimates an unknown value on the basis of two known values that surround it.

In this case, the 7th value is 37 and the 8th value is 39. The 6th decile is: $P_{60} = 37 + 0.4 (0.2 \text{ times the linear distance between 37 and 39})$. $P_{60} = 37.4$

7. Measures of Dispersion

In this segment we look at measures that tell us how spread out or dispersed our data might be.

7.1 The Range

The range is the difference between the maximum and minimum values in a data set. It is expressed as:

Range = Max value – Min Value

If the annual returns data is: 10%, -5%, 10%, 25%. What is the range?

Here the maximum return is 25% and the minimum return is -5%. The range is $25\% - (-5\%) = 30\%$.

7.2 The Mean Absolute Deviation

It is the average of the absolute values of deviations from the mean. It is expressed as:

$$MAD = \left[\sum_{i=1}^n |X_i - \bar{X}| \right] / n$$

where: \bar{X} is the sample mean and n is the number of observations in the sample.

Example

Consider the following data set: 8, 12, 10, 8 and 5. Calculate the mean absolute deviation.

Solution:

$$\bar{X} = (8 + 12 + 10 + 8 + 5) / 5 = 8.6$$

$$MAD = \frac{|8 - 8.6| + |12 - 8.6| + |10 - 8.6| + |8 - 8.6| + |5 - 8.6|}{5}$$

$$MAD = \frac{0.6 + 3.4 + 1.4 + 0.6 + 3.6}{5} = 1.92$$

7.3 Population Variance and Population Standard Deviation

Population variance is the mean of the squared deviations from the mean. The population variance is computed using all members of a population. It is expressed as:

$$\sigma^2 = \sum_{i=0}^N (X_i - \mu)^2 / N$$

where: μ is the population mean and N is the size of the population

Population standard deviation is defined as the positive square root of the population variance.

Example

Calculate the population variance and standard deviation for this dataset: 2%, 5%, 4%, 7%, 8%, 8%, 12%, 10%, 8%, and 5%.

Solution:

$$\sigma^2 =$$

$$\frac{[(2 - 6.9)^2 + (5 - 6.9)^2 + (4 - 6.9)^2 + (7 - 6.9)^2 + (8 - 6.9)^2 + (8 - 6.9)^2 + (12 - 6.9)^2 + (10 - 6.9)^2 + (8 - 6.9)^2 + (5 - 6.9)^2]}{10}$$

$$\sigma^2 = 7.89\%$$

$$\text{Population standard deviation } (\sigma) = \sqrt{7.89} = 2.81\%$$

7.4 Sample Variance and Sample Standard Deviation

Sample variance applies when we are dealing with a subset, or sample, of the total population. It is expressed as:

$$s^2 = \sum_{i=0}^n (X_i - \bar{X})^2 / (n - 1)$$

where: \bar{X} is the sample mean and n is the number of observations in the sample.

Sample standard deviation is defined as the positive square root of the sample variance.

Example

Calculate the sample variance for the following data set: 8, 12, 10, 8 and 5.

Solution:

$$s^2 = \frac{[(8 - 8.6)^2 + (12 - 8.6)^2 + (10 - 8.6)^2 + (8 - 8.6)^2 + (5 - 8.6)^2]}{5 - 1}$$

$$s^2 = 6.80\%$$

The sample standard deviation is the positive square root of the sample variance. For the sample data given above, $s = \sqrt{6.80} = 2.61\%$

Using a financial calculator to calculate variance and standard deviations

The population and sample standard deviation can easily be computed using a financial calculator. Assume the following data set: 10%, -5%, 10%, 25%, the calculator key strokes are shown below:

Keystrokes	Description	Display
[2nd] [DATA]	Enters data entry mode	
[2nd] [CLR WRK]	Clears data register	X01
10 [ENTER]		X01 = 10
[↓] [↓] 5+/- [ENTER]		X02 = -5
[↓] [↓] 10 [ENTER]		X03 = 10
[↓] [↓] 25 [ENTER]		X04 = 25
[2nd] [STAT] [ENTER]	Puts calculator into stats mode	
[2nd] [SET]	Press repeatedly till you see →	1-V
[↓]	Number of data points	N = 4
[↓]	Mean	X = 10
[↓]	Sample standard deviation	Sx = 12.25
[↓]	Population standard deviation	σx = 10.61

Notice that the calculator gives both the sample and the population standard deviation. On the exam we will have to determine whether we are dealing with population or sample data and choose the appropriate value.

7.5 Semivariance, Semideviation, and Related Concepts

Instructor's Note: Semivariance and semideviation are not emphasized in the learning outcomes and have a very low probability of being tested on the Level I exam. Nevertheless, a brief explanation is given below.

Variance and standard deviation of returns take account of returns above and below the mean, but often investors are concerned only with downside risk, for example returns below the mean. As a result, analysts have developed semivariance, semideviation and related dispersion measures that focus on downside risk. Semivariance is defined as the average squared deviation below the mean. Semideviation is the positive square root of semivariance.

7.6 Chebyshev's Inequality

According to Chebyshev's inequality, the proportion of the observations within k standard deviations of the arithmetic mean is at least: $1 - \frac{1}{k^2}$ for all $k > 1$.

To find out what percent of the observations must be within two standard deviations of the mean we simply plug into the formula and get: $1 - \frac{1}{2^2} = 1 - \frac{1}{4} = 0.75 = 75\%$. Hence, at least 75% of the data will be between two standard deviations of the mean.

Chebyshev's inequality can be used to measure maximum amount of dispersion, regardless of the shape of the distribution. Notice that here we do not make any assumptions about whether the distribution is normal or not normal. This inequality applies across all distributions.

7.7 Coefficient of Variation

Coefficient of variation expresses how much dispersion exists relative to the mean of a distribution and allows for direct comparison of dispersion across different data sets. It is used in investment analysis to compare relative risks. When evaluating investments, a lower value is better. Coefficient of variation is expressed as:

$$CV = \frac{s}{\bar{X}}$$

where: s = sample standard deviation of a set of observations and \bar{X} = sample mean

Example

Investment A has a mean return of 7% and a standard deviation of 5%. Investment B has a mean return of 12% and a standard deviation of 7%. Calculate the coefficients of variation.

Solution

The coefficients of variation can be calculated as follows:

$$CV_A = \frac{5\%}{7\%} = 0.71$$

$$CV_B = \frac{7\%}{12\%} = 0.58$$

This metric shows that Investment A is riskier than Investment B.

7.8 The Sharpe Ratio

The Sharpe ratio is the ratio of excess return to standard deviation of return for a portfolio. Excess return refers to the return above the risk-free rate. When evaluating investments, a higher value is better. The formula for calculating the Sharpe ratio is:

$$S_p = \frac{\bar{R}_p - \bar{R}_F}{s_p}$$

where:

\bar{R}_p = Mean return to the portfolio

\bar{R}_F = Mean return to a risk-free asset

s_p = Standard deviation of return on the portfolio

Example

The table below provides data for two portfolios. Given that the mean annual risk free rate is 10.5%, which portfolio has the higher Sharpe ratio?

Portfolio	Arithmetic mean return (%)	Variance of (%)
Portfolio A	16.4%	4.9%
Portfolio B	12.6%	3.5%

Solution:

$$\text{Portfolio A: } \frac{16.4 - 10.5}{\sqrt{4.9}} = 2.665$$

$$\text{Portfolio B: } \frac{12.6 - 10.5}{\sqrt{3.5}} = 1.122$$

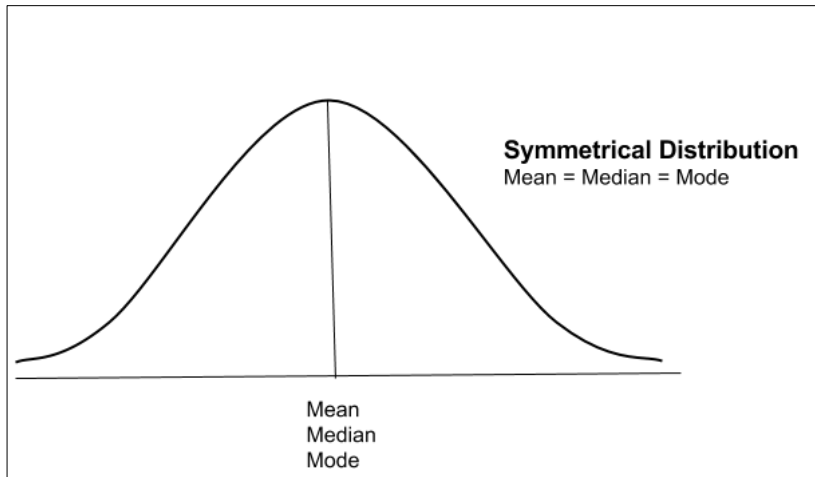
Portfolio A offers a higher excess return per unit of risk relative to Portfolio B.

8. Symmetry and Skewness in Return Distributions

Symmetrical distribution

A distribution is said to be symmetrical when the distribution on either side of the mean is a mirror image of the other.

In a normal distribution, mean = median = mode.

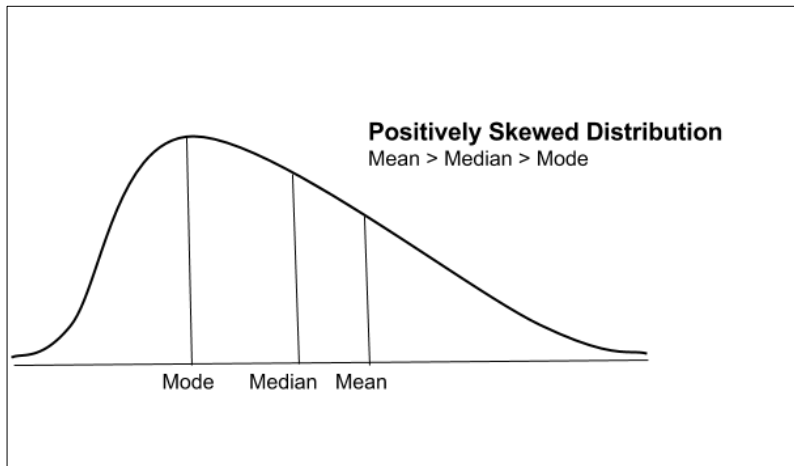


If a distribution is non-symmetrical, it is said to be skewed. Skewness can be negative or positive.

Positively skewed distribution

A positively skewed distribution has a long tail on the right side, which means that there will be frequent small losses and few large gains.

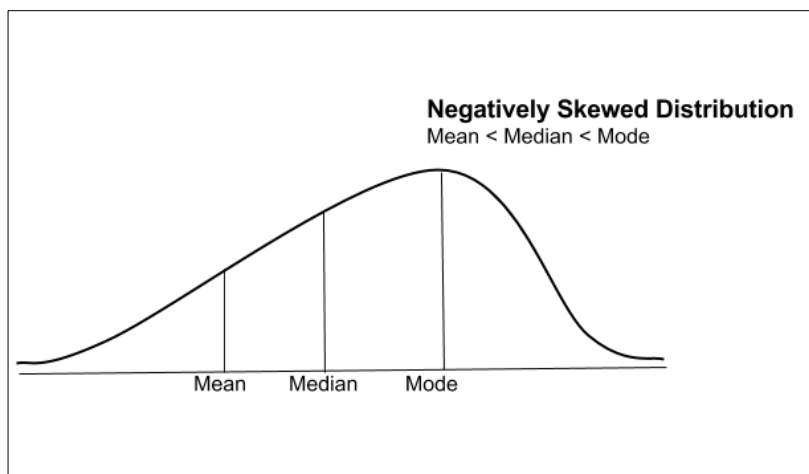
Here the mean > median > mode. The extreme values affect the mean the most which is pulled to the right. They affect the mode the least.



Negatively skewed distribution

A negatively skewed distribution has a long tail on the left side, which means that there will be frequent small gains and few large losses.

Here the mean < median < mode. The extreme values affect the mean the most which is pulled to the left. They affect the mode the least.

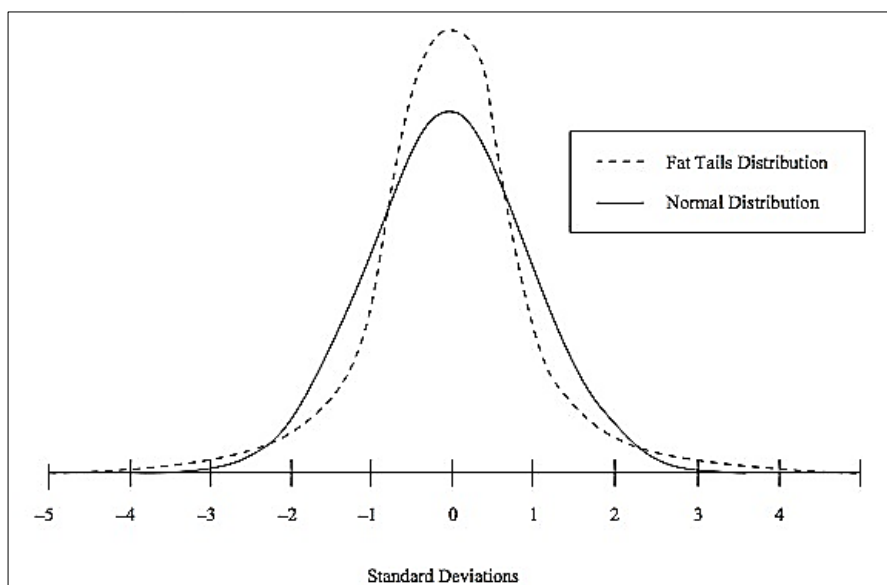


9. Kurtosis in Return Distributions

Kurtosis is a measure of the degree to which a distribution is more or less peaked than a normal distribution, which has a kurtosis of 3.

Excess kurtosis = kurtosis - 3. An excess kurtosis with an absolute value greater than one is considered significant.

- A **leptokurtic** distribution is more peaked and has fatter tails than a normal distribution.
- A **platykurtic** distribution is less peaked and has thinner tails than a normal distribution.
- A **mesokurtic** distribution is identical to a normal distribution.



10. Using Geometric and Arithmetic Means

The geometric mean is appropriate to measure past performance over multiple periods.

Example

The portfolio returns for the past two years were 100% in year 1 and -50% in year 2. What was the mean return?

Solution:

Past return = geometric mean = $(2 \times 0.5)^{0.5} - 1 = 0\%$

The arithmetic mean is appropriate for forecasting single period returns.

Example

Two possible returns for the next year are 100% and -50%. What is the expected return?

Solution:

Expected return = Arithmetic mean = $(100 - 50)/2 = 25\%$

Summary

LO.a: Distinguish between descriptive statistics and inferential statistics, between a population and a sample, and among the types of measurement scales.

The Nature of Statistics

- Descriptive statistics: It refers to how large data sets can be summarized effectively to describe their important characteristics.
- Inferential statistics: It refers to making forecasts, estimates or judgments about a large data set based on a small representative set.

Population and Sample

- Population: It includes all members of a particular group.
- Sample: It is a subset drawn from a population.

Measurement Scales: Data can be measured using the following scales:

- Nominal scale: They put data in categories but do not rank them.
- Ordinal scale: Nominal scale + data can be ranked with respect to some characteristic.
- Interval scale: Ordinal scale + the differences in the data values are meaningful.
- Ratio scale: Interval scale + the ratios of value, such as twice or half as much are meaningful.

LO.b: Define a parameter, a sample statistic, and a frequency distribution.

Parameter: A descriptive measure of a population is called a parameter.

Sample statistic: A descriptive measure of a sample is called a sample statistic.

Frequency distribution: A frequency distribution is a tabular display of data categorized into a relatively small number of intervals or classes. It allows us to evaluate how data is distributed.

LO.c: Calculate and interpret relative frequencies and cumulative relative frequencies, given a frequency distribution.

Relative frequency: It is calculated as the absolute frequency of an interval divided by the total number of observations.

Cumulative relative frequency: For an interval, it is calculated as the sum of the relative frequencies of all intervals lower than and including that interval.

LO.d: Describe the properties of a data set presented as a histogram or a frequency polygon.

Histogram is a bar chart of data that has been grouped together into a frequency distribution. The height of each bar is equal to the absolute frequency of each interval.

A frequency polygon plots the midpoints of each interval on the X-axis and the absolute frequency of that interval on the Y-axis. Each point is then connected with a straight line.

LO.e: Calculate and interpret measures of central tendency, including the population mean, sample mean, arithmetic mean, weighted average or mean, geometric mean, harmonic mean, median, and mode.

Arithmetic mean is simply the sum of all the observations divided by the total number of observations.

Median is the midpoint of a data set that has been sorted from largest to smallest or vice-versa.

Mode is the value that occurs most frequently in a data set.

The geometric mean is used to calculate compound growth rate. It is computed as:

$$R_G = [(1 + R_1) (1 + R_2) \dots (1 + R_n)]^{1/n} - 1$$

In a weighted mean, different observations are given different weights as per their proportional influence on the mean. It is computed as:

$$\bar{X}_w = \sum_{i=1}^n w_i X_i$$

Harmonic mean is used to find average purchase price for equal periodic investments. It is computed as:

$$X_H = n / \sum_{i=1}^n \left(\frac{1}{X_i} \right)$$

LO.f: Calculate and interpret quartiles, quintiles, deciles, and percentiles.

A quantile is a value at or below which a stated fraction of the data lies. Some examples of quantiles include:

- **Quartiles:** The distribution is divided into quarters.
- **Quintiles:** The distribution is divided into fifths.
- **Deciles:** The distribution is divided into tenths.
- **Percentile:** The distribution is divided into hundredths.

The formula for the position of a percentile in a data set with n observations sorted in ascending order is:

$$L_y = (n+1) y / 100$$

LO.g: Calculate and interpret 1) a range and a mean absolute deviation and 2) the variance and standard deviation of a population and of a sample.

Range is the difference between the maximum and minimum values in a data set. It is calculated as:

$$\text{Range} = \text{Maximum value} - \text{Minimum value}$$

Mean absolute deviation (MAD) is the average of the absolute values of deviations from the mean. It is expressed as:

$$\text{MAD} = \left[\sum_{i=1}^n |X_i - \bar{X}| \right] / n$$

Variance is defined as the mean of the squared deviations from the arithmetic mean. It is calculated as:

$$\text{Population Variance } \sigma^2 = \sum_{i=0}^N (X_i - \mu)^2 / N$$

$$\text{Sample Variance } s^2 = \sum_{i=0}^n (X_i - \bar{X})^2 / (n - 1)$$

Standard deviation is the positive square root of the variance. It is often used as a measure of risk.

LO.h: Calculate and interpret the proportion of observations falling within a specified number of standard deviations of the mean using Chebyshev's inequality.

According to Chebyshev's inequality, the proportion of the observations within k standard deviations of the arithmetic mean is at least $1 - 1/k^2$ for all $k > 1$.

LO.i: Calculate and interpret the coefficient of variation and the Sharpe ratio.

Coefficient of variation measures the risk per unit of return. When evaluating investments, a lower value is better. It is expressed as:

$$\text{CV} = \frac{s}{\bar{X}}$$

Sharpe ratio measures excess return per unit of risk. When evaluating investments, a higher value is better. It is expressed as:

$$S_p = \frac{\bar{R}_p - \bar{R}_F}{s_p}$$

LO.j: Explain skewness and the meaning of a positively or negatively skewed return distribution.

Skew describes the degree to which a distribution is not symmetric about its mean. A return distribution with positive skewness has frequent small losses and a few extreme gains. A return distribution with negative skewness has frequent small gains and a few extreme losses. Zero skewness indicates a symmetric distribution of returns.

LO.k: Describe the relative locations of the mean, median, and mode for a unimodal, nonsymmetrical distribution.

For the positively skewed unimodal distribution, the mode is less than the median, which is less than the mean. For the negatively skewed unimodal distribution, the mean is less than the median, which is less than the mode.

LO.l: Explain measures of sample skewness and kurtosis.

Kurtosis is a measure of the degree to which a distribution is more or less peaked than a normal distribution, which has a kurtosis of 3.

Excess kurtosis = kurtosis - 3. An excess kurtosis with an absolute value greater than one is considered significant.

- A **leptokurtic** distribution is more peaked and has fatter tails than a normal distribution.
- A **platykurtic** distribution is less peaked and has thinner tails than a normal distribution.
- A **mesokurtic** distribution is identical to a normal distribution.

LO.m: Compare the use of arithmetic and geometric means when analyzing investment returns.

To estimate the average returns over more than one period (time series data), you should use the geometric mean because it captures how the total returns are linked over time. If you want to estimate the average return of multiple investments over a one-period horizon (cross-sectional data), you should use the arithmetic mean.

Practice Questions

1. An analyst rates stocks as underweight, market weight or overweight, referring to the suggested weighting of the stock in a portfolio. The measurement scale used by the analyst can *best* be described as:
 - A. nominal scale.
 - B. ordinal scale.
 - C. interval scale.
2. A parameter describes the characteristic of a:
 - A. population.
 - B. sample.
 - C. population and a sample.
3. For the following frequency distribution, the number of intervals, the sample size and the relative frequency of the third interval are *closest* to:

Returns	Frequency
-10% up to 0%	4
0% up to 10%	13
10% up to 20%	5
20% up to 30%	3

- A. 4, 25, 20%.
 - B. 1, 20, 10%.
 - C. 3, 30, 15%.
4. The following ten observations are a sample drawn from a normal population: 20, 15, 12, 6, 4, -11, 19, 14, -3, and 19. The arithmetic mean of the sample is *closest* to:
 - A. 8.6.
 - B. 9.5.
 - C. 10.2.
 5. A portfolio has the following annual returns: 6%, 10%, -5%, 0%. The geometric mean across the four-year period is *closest* to:
 - A. 2.37%.
 - B. 2.48%.
 - C. 2.59%.
 6. Consider the following set of numbers:

-20	-11	-3	0	1
3	5	6	9	10

12	18	21	23	25
28	39	40	54	60

The median value of the above items is *closest* to:

- A. 10.
- B. 11.
- C. 12.

7. Consider the following set of numbers:

20	24	21	35	20	24
24	22	20	26	28	22
19	21	20	21	22	28

The mode of the sample is closet to:

- A. 20.
- B. 21.
- C. 22.

8. Following data is provided on returns of a series of funds:

Fund	Returns (%)
Fund 1	6.15%
Fund 2	7.25%
Fund 3	8.95%
Fund 4	9.65%
Fund 5	10.35%
Fund 6	11.10%
Fund 7	12.35%
Fund 8	13.55%
Fund 9	14.05%
Fund 10	15.75%
Fund 11	16.95%
Fund 12	17.25%
Fund 13	18.30%

The value of the second quintile is *closest* to:

- A. 10.35%
- B. 10.80%.
- C. 11.30%.

9. The annual returns of a portfolio are given below:

Year	Portfolio return
Year 1	6.5%

Year 2	8.2%
Year 3	10.5%
Year 4	-5.4%
Year 5	7.7%

The portfolio's range and mean absolute deviation for the five-year period are *closest* to:

- A. 15.9% and 5.1%.
- B. 5.1% and 4.36%.
- C. 15.9% and 4.36%.

10. The following observations are drawn from an approximately normal population:

Observation	1	2	3	4	5
Value	11	8	-6	4	-5

The sample standard deviation is *closest* to:

- A. 6.83 and 46.65
- B. 7.64 and 58.37
- C. 7.64 and 2.76

11. A sample of 240 observations is randomly selected from a population. The mean of the sample is 120 and the standard deviation is 8. Based on Chebyshev's inequality, the endpoints of the interval that must contain at least 75% of the observations are *closest* to:

- A. 96 and 144.
- B. 112 and 128.
- C. 104 and 136.

12. George Baker, an equity fund manager has the following information about a common stock portfolio:

Arithmetic mean return	11.8%
Geometric mean return	10.6%
Portfolio beta	1.2
Risk-free rate of return	4.25%
Variance of returns	196

From the given information, the coefficient of variation is *closest* to:

- A. 1.32.
- B. 1.24.
- C. 1.18.

13. Following information about three portfolios is given:

Portfolio	Mean return on portfolio (%)	Standard deviation of the returns (%)
A	15	28
B	12	19
C	10	7

If the risk-free rate is 4%, which portfolio has the *highest* Sharpe ratio?

- A. A.
- B. B.
- C. C.

14. Which of the following is *most likely* to be true for a positively skewed distribution?

- A. Mode > Median > Mean.
- B. Median > Mode > Mean.
- C. Mean > Median > Mode.

15. A distribution less peaked than the normal distribution is *best* described as being:

- A. platykurtic.
- B. mesokurtic.
- C. leptokurtic.

Solutions

1. B is correct. This is an ordinal scale. The rating of underweight, market weight or overweight orders the rated securities in terms of levels of expected performance.
2. A is correct. A parameter describes the characteristic of a population, while a sample statistic describes the characteristic of a sample.
3. A is correct. An interval is the set of return values that an observation falls within. There are four intervals. The sample size is the sum of all of the frequencies in the distribution: $4 + 13 + 5 + 3 = 25$. The relative frequency is found by dividing the frequency of the interval by the total number of frequencies: $5/25 = 20\%$.
4. B is correct. The sum of the ten numbers is 95. Dividing by 10 gives the mean of 9.5.
5. C is correct. The geometric mean return is calculated as $[(1 + 0.06) \times (1 + 0.1) \times (1 - 0.05) \times (1 + 0.00)]^{1/4} - 1 = 0.0259 \approx 2.59\%$.
6. B is correct. The median is the value of the middle item of a set of items. The total number of items are 20, hence the median is the value between 10th and 11th observation. The value of the 10th item is 10; the value of the 11th item is 12. The average of 10 and 12 is 11.
7. A is correct. The mode is the most frequent value in the set of items and thus is equal to 20.
8. B is correct. Quintiles divide data into five parts. Hence the first quintile corresponds to the 20th percentile and the second quintile corresponds to the 40th percentile. The location can be determined using: $L_y = (n + 1) \times (y / 100) \rightarrow L_{40} = (13+1) \times (40/100) = 5.6$. The value corresponding to location 5 (Fund 5) is 10.35%. The value corresponding to location 6 (Fund 6) is 11.10%. The approximate value corresponding to location 5.6 can be estimated using linear interpolation: $10.35\% + (0.6 \times (11.10\% - 10.35\%)) = 10.80\%$
9. C is correct. Range = Highest value – Lowest value = $10.5\% - (-5.4\%) = 15.9\%$
To calculate MAD, first compute the mean: $(6.5 + 8.2 + 10.5 - 5.4 + 7.7) / 5 = 5.5\%$ and compute MAD, $(|6.5 - 5.5| + |8.2 - 5.5| + |10.5 - 5.5| + |-5.4 - 5.5| + |7.7 - 5.5|) / 5 = 4.36\%$.
10. B is correct. The sample standard deviation can be found as:

Keystrokes	Explanation	Display
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Keystrokes	Explanation	Display
[2nd] [DATA]	Enter data entry mode	
[2nd] [CLR WRK]	Clear data registers	X01
11 [ENTER]		X01 = 11
[↓] [↓] 8 [ENTER]		X02 = 8
[↓] [↓] 6 +/- [ENTER]		X03 = -6
[↓] [↓] 4 [ENTER]		X04 = 4
[↓] [↓] 5 +/- [ENTER]		X05 = -5
[2nd] [STAT]	Puts calculator into stats	
[2nd] [SET]	Press repeatedly till you	1-V
[↓]	Number of data points	N = 5
[↓]	Mean	X = 2.4
[↓]	Sample standard	Sx = 7.64
[↓]	Population standard	$\sigma_x = 6.83$

The sample variance is simply the square of sample standard deviation: $7.64^2 = 58.37$.

11. C is correct. According to Chebyshev's inequality, the proportion of the observations within k standard deviations of the arithmetic mean is at least $1 - 1/k^2$ for all $k > 1$. For $k = 2$, that proportion is $1 - 1/2^2$, which is 75%. The lower endpoint is, therefore, the mean (120) minus 2 times 8 (the standard deviation) and the upper endpoint is 120 plus 2 times 8.

$$120 - (2 \times 8) = 104; 120 + 2(8) = 136.$$

12. C is correct. The coefficient of variation is: Standard deviation of return / Mean return = $\sqrt{196} / 11.8 = 1.18$

13. C is correct. The Sharpe ratio is defined as $S_p = (R_p - R_F) / s_p$

$$S_A = (15 - 4) / 28 = 0.3928$$

$$S_B = (12 - 4) / 19 = 0.4210$$

$$S_C = (10 - 4) / 7 = 0.8571$$

$$\text{Sharpe Ratio} = \frac{R_p - R_F}{S_p} \text{ excess returns per unit of risk. A higher number is preferred.}$$

14. C is correct. For a positively skewed distribution, the mean is greater than the median, which is greater than the mode.

15. A is correct. Platykurtic describes a distribution that is less peaked than a normal distribution.

Leptokurtic describes a distribution that is more peaked than the normal distribution.

Mesokurtic is a distribution as peaked as the normal distribution.

R09 Probability Concepts

1. Introduction

Since many investment decisions are made in an environment of uncertainty, it is essential for portfolio managers and investment managers to have a fundamental grasp of probability concepts. In this reading, we will focus on:

- Definitions and rules related to probability.
- Expected value and variance.
- Covariance and correlation.

2. Probability, Expected Value, and Variance

Fundamental Concepts

Random variable is an uncertain quantity/number. For example, when you roll a die, the result is a random variable.

An **outcome** is the observed value of a random variable. For example, if you roll a 2, it is an outcome.

An **event** can be a single outcome or a set of outcomes. For example, you can define an event as rolling a 2 or rolling an even number.

Mutually exclusive events are events that cannot happen at the same time. For example, rolling a 2 and rolling a 3 are examples of mutually exclusive events. They cannot happen at the same time.

Exhaustive events are those that cover all possible outcomes. For example, 'rolling an even number' or 'rolling an odd number' are exhaustive events. They cover all possible outcomes.

The **two defining properties of probability** are:

- The probability of any event has to be between 0 and 1.
- The sum of the probabilities of mutually exclusive and exhaustive events is equal to 1.

Ways of Estimating Probability

The methods of estimating probabilities are:

- **Empirical probability:** Based on analyzing the frequency of an event's occurrence in the past.
- **A priori probability:** Based on formal reasoning and inspection rather than personal judgment.
- **Subjective probability:** Informed guess based on personal judgment.

Empirical and a priori probabilities are often grouped as objective probabilities because they do not vary from person to person.

Probability Stated as Odds

Odds for an event are defined as the probability of the event occurring to the probability of the event not occurring. Odds for E = $P(E) / [1 - P(E)]$.

Given odds for E of “a to b”, the implied probability of E is $a / (a + b)$.

Example

If the probability of an event is 0.2, what are the odds of it occurring? Alternatively, if the odds are 1 to 4, what is the probability of this event?

Solution:

The odds of the event occurring are $= \frac{0.2}{0.8} = 1/4$. This is stated as odds of 1 to 4.

Given the odds, the probability of the event occurring is $\frac{1}{1+4} = \frac{1}{5} = 0.20$.

Odds against an event are defined as the probability of the event not occurring to the probability of the event occurring. Odds against E = $[1 - P(E)] / P(E)$.

Give odds against E of “a to b”, the implied probability of E is $b / (a + b)$.

Example

If $P(E) = 0.2$, what are the odds against the event occurring? If the odds against an event are 4 to 1, what is the probability of the event?

Solution:

$P(E) = \frac{0.8}{0.2} = \frac{4}{1}$. Hence the odds against E are 4 to 1.

Given the odds against an event, the probability of the event is $\frac{1}{4+1} = 0.2$

Conditional v/s Unconditional probabilities

Unconditional probability is the probability of an event occurring irrespective of the occurrence of other events. It is denoted as $P(A)$.

Conditional probability is the probability of an event occurring given that another event has occurred. It is denoted as $P(A|B)$, which is the probability of event A given that event B has occurred.

Joint Probability and Multiplication Rule

Multiplication rule is used to determine the joint probability of two events. It is expressed as:

$$P(AB) = P(A|B) P(B)$$

Rearranging the equation we get the formula for computing conditional probabilities:

$$P(A|B) = P(AB) / P(B)$$

Example

$P(\text{interest rates will decrease}) = P(D) = 40\%$

$P(\text{stock price increases}) = P(S)$

$P(\text{stock price will increase given interest rates decrease}) = P(S|D) = 70\%$

Compute probability of a stock price increase **and** an interest rate decrease.

Solution:

$P(SD) = P(S|D) \times P(D) = 0.7 \times 0.4 = 0.28 = 28\%$

Addition Rule for Probabilities

Addition rule is used to determine the probability that at least one of the events will occur. It is expressed as:

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

Example

$P(\text{price of A increases}) = P(A) = 0.5$

$P(\text{price of B increases}) = P(B) = 0.7$

$P(\text{price of A and B increases}) = P(AB) = 0.3$

Compute the probability that the price of stock A **or** the price of stock B increases.

Solution

$P(A \text{ or } B) = 0.5 + 0.7 - 0.3 = 0.9$

Independent and Dependent Events

If the occurrence of one event does not influence the occurrence of the other event, then the two events are called **independent events**.

i.e. $P(A|B) = P(A)$ or $P(B|A) = P(B)$

Multiplication rule for independent events: $P(AB) = P(A) P(B)$

Addition rule for independent events: $P(A \text{ or } B) = P(A) + P(B) - P(AB)$. (The addition rule does not change.)

If the probability of an event is affected by the occurrence of another event then it is called a **dependent event**.

Total Probability Rule

The total probability rule is used to calculate the unconditional probability of an event, given conditional probabilities.

In investment analysis, we often formulate a set of mutually exclusive and exhaustive scenarios and then estimate the probability of a particular event. For example, let's say that

we have two scenarios S and non-S that are mutually exclusive and exhaustive.

According to the total probability rule:

$$P(A) = P(AS) + P(AS^c)$$

Using the multiplication rule we get,

$$P(A) = P(A|S) P(S) + P(A|S^c) P(S^c)$$

If we have more than two scenarios, we can generalize this equation to:

$$P(A) = P(AS_1) + P(AS_2) + \dots + P(AS_n) = P(A|S_1) P(S_1) + P(A|S_2) P(S_2) + \dots + P(A|S_n) P(S_n)$$

Expected Value of a Random Variable

The expected value of a random variable can be defined as the probability-weighted average of the possible outcomes of the random variable. For a random variable X, the expected value of X is denoted as E(X) and is calculated as:

$$E(X) = \sum_{i=1}^n P(X_i) X_i$$

where:

X_i = One of n possible outcomes of the random variable X

$P(X_i)$ = Probability of X_i

Variance of a Random Variable

The expected value is our forecast, but we cannot count on the individual forecast being realized. This is why we need to measure the risk we face. Variance and standard deviation are examples of how we can measure this risk. The variance of a random variable is the probability-weighted sum of the squared differences between each possible outcome and the expected value of the random variable. It is expressed as:

$$\sigma^2(X) = \sum_{i=1}^n P(X) [X - E(X)]^2$$

Variance is a number greater than or equal to 0 because it is the sum of squared terms. If variance is 0, there is no dispersion or risk. The outcome is certain and the quantity X is not random at all. Standard deviation is the positive square root of variance.

We can calculate the expected value and variance of a random variable using a financial calculator as shown below:

Example

A project's cash flow for the upcoming year depends on the state of the economy, as shown in the table below. What is the variance of the cash flow? What is the standard deviation?

State of Economy	Probability	Cash Flow
Good	0.3	50
Average	0.5	40
Weak	0.2	20

Solution:

Using a financial calculator:

Keystrokes	Explanation	Display
[2nd] [DATA]	Enters data entry mode	
[2nd] [CLR WRK]	Clears data register	X01
50 [ENTER]	1 st possible value of random variable	X01 = 50
[↓] 30 [ENTER]	Probability of 30% for X01	Y01 = 30
[↓] 40 [ENTER]	2 nd possible value of random variable	X02 = 40
[↓] 50 [ENTER]	Probability of 50% for X02	Y02 = 50
[↓] 20 [ENTER]	3 rd possible value of random variable	X03 = 20
[↓] 20 [ENTER]	Probability of 20% for X03	Y03 = 20
[2nd] [STAT]	Puts calculator into stats mode	
[2nd] [SET]	Press repeatedly till you see →	1-V
[↓]	Total number of entries	N = 100
[↓]	Expected value of random variable	X = 39
[↓]	Sample standard deviation	Sx = 10.49
[↓]	Population standard deviation	$\sigma_x = 10.44$

We can then square the population standard deviation of 10.44 to get the variance i.e. $10.44^2 = 109.00$

Total Probability Rule for Expected Value

Just like the total probability rule states unconditional probabilities in terms of conditional probabilities, the total probability rule for expected values states unconditional expected values in terms of conditional expected values.

$$E(X|S) = P(X_1|S) X_1 + P(X_2|S) X_2 + \dots + P(X_n|S) X_n$$

Instructor's Note

Notice that this formula is exactly similar to the total probability rule formula.

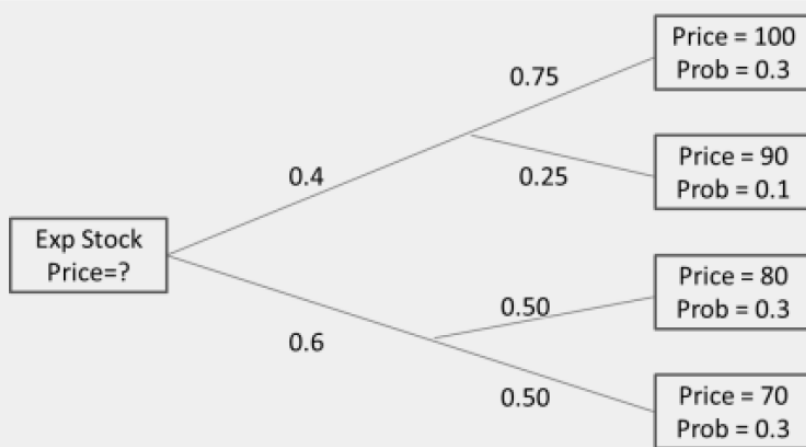
$$P(A) = P(A|S_1) P(S_1) + P(A|S_2) P(S_2) + \dots + P(A|S_n) P(S_n)$$

Example

What is the expected price of a stock at the end of the current period given the following information: probability that interest rates will decline = 0.4. If interest rates decline there is a 75% chance that stock price will be \$100 versus a 25% chance that the stock price will be \$90. If interest rates do not decline there is a 50% chance that the stock price will be \$80 versus a 50% chance that stock price will be \$70.

Solution:

We can plot the probabilities using a tree diagram.



Consider the first node (top right). It refers to the probability that the stock price will be \$100 given a decline in interest rates. We can calculate the probability of that happening by multiplying the probability of a decline in interest rates (0.4) by the probability of the stock price being \$100 if that happens (0.75). This gives us an conditional probability of 0.30. In short, it is the joint probability of the stock price being \$100 given a decline in interest rates. Similarly, probabilities are calculated for each of the other three nodes. We can then calculate:

$$E(\text{Price} \mid \text{decline in interest rates}) = 0.75 (\$100) + 0.25 (\$90) = \$97.50$$

$$E(\text{Price} \mid \text{no decline in interest rates}) = 0.50 (\$80) + 0.50 (\$70) = \$75.00$$

Now we use the total probability rule for expected value of stock price at the end of the current period:

$$E(\text{Price}) = E(\text{Price} \mid \text{decline in interest rates}) P(\text{decline in interest rates}) + E(\text{Price} \mid \text{no decline in interest rates}) P(\text{no decline in interest rates})$$

$$E(\text{Price}) = \$97.50 (0.40) + \$75.00 (0.60)$$

$$E(\text{Price}) = \$84.00$$

3. Portfolio Expected Return and Variance of Return

Expected Return

A portfolio's expected return can be calculated as:

$$E(R_P) = w_1E(R_1) + w_2E(R_2) + \dots + w_nE(R_n)$$

where:

w_n = portfolio weight of n^{th} security in the portfolio

R_n = expected return of n^{th} security in the portfolio

n = number of securities in the portfolio

We will discuss portfolio expected return and variance of return using a two-stock portfolio.

Example

40% of the portfolio is invested in Stock A and 60% is invested in Stock B. As shown in the table below, the expected return of each stock depends on the economic scenario.

Scenario	P(Scenario)	Expected returns of A	Expected returns of B
Recession	0.25	2%	4%
Normal	0.50	8%	10%
Boom	0.25	12%	16%

This information can also be presented as a joint probability function of A's and B's returns:

	$R_B = 4\%$	$R_B = 10\%$	$R_B = 16\%$
$R_A = 2\%$	0.25	0	0
$R_A = 8\%$	0	0.50	0
$R_A = 12\%$	0	0	0.25

Row 1 and Column 1 represent the returns of A and B respectively. The other cells contain probabilities. Calculate the expected return of A and B.

Solution:

Given the data presented above:

The expected return of A is: $0.25 \times 2 + 0.50 \times 8 + 0.25 \times 12 = 7.5\%$.

The expected return of B is: $0.25 \times 4 + 0.50 \times 10 + 0.25 \times 16 = 10\%$.

Expected return of the portfolio = weight of A in the portfolio \times expected return of A + weight of stock B in the portfolio \times expected return of B = $0.4 \times 7.5 + 0.6 \times 10 = 9\%$

The expected portfolio return is 9%. As the term implies, this is the expected return. The actual return will vary around 9%. The amount of variability is measured by the variance. In order to determine the variance of return, we must first calculate the covariance.

Covariance

Covariance tells us how movements in a random variable vary with movements in another

random variable, whereas variance tells us how a random variable varies with itself. Assume there are two random variables R_i and R_j . the covariance between R_i and R_j (used to measure how they move together) is given by:

$$\text{Cov}(R_i, R_j) = E([R_i - ER_i][R_j - ER_j])$$

where:

ER_i = expected return for variable R_i

ER_j = expected return for variable R_j

Example

Continuing with our previous example, calculate the covariance of returns between A and B.

Solution:

Say R_i represents the return on A and R_j represents the return on B, we have already calculated the expected returns of A and B as 7.5% and 10% respectively. The covariance of returns is:

$$\begin{aligned} &E[(R_i - 7.5)(R_j - 10)] \\ &= 0.25(2\% - 7.5\%)(4\% - 10\%) + 0.5(8\% - 7.5\%)(10\% - 10\%) + 0.25(12\% - 7.5\%)(16\% - 10\%) \\ &= 0.000825 + 0 + 0.000675 = 0.0015 \end{aligned}$$

Correlation

The problem with covariance is that it can vary from negative infinity to positive infinity which makes it difficult to interpret. To address this problem, we use another measure called correlation. Correlation is a standardized measure of the linear relationship between two variables with values ranging between -1 and +1.

- A correlation of 0 (uncorrelated variables) indicates an absence of any linear (straight-line) relationship between the variables.
- A correlation of +1 indicates a perfect positive relationship.
- A correlation of -1 indicates a perfect negative relationship.

It is computed as:

$$\rho(R_i, R_j) = \text{Cov}(R_i, R_j) / \sigma(R_i) \sigma(R_j)$$

We will now apply this formula to calculate the correlation between the returns of A and B from our example. We have already shown that the covariance of returns is 0.0015. In order to calculate the correlation, we need the standard deviation of A and B. Using a financial calculator, we can determine that the standard deviation of A is 0.0357 and the standard deviation of B is 0.0424. The correlation, $\rho(A, B) = \frac{\text{Cov}(A, B)}{\sigma(A)\sigma(B)} = 0.0015 / (0.0357 \times 0.0424) = 0.99$.

The correlation of 0.99 (almost 1) implies a very strong positive relationship between the returns of A and B. This is more meaningful than the covariance number of 0.0015 which tells us that there is a positive relationship between the returns of A and B but does not give a sense for the strength of the relationship.

Variance of returns

Once we know the covariance, we can calculate the variance of a portfolio using this formula:

$$\sigma^2(R_p) = w_1^2 \sigma_1^2 (R_1) + w_2^2 \sigma_2^2 (R_2) + 2w_1 w_2 \text{Cov} (R_1 R_2)$$

Example

Continuing with our example, the variance of the portfolio is

- Weight of the first asset, $w_1 = 0.40$
- Weight of the second asset, $w_2 = 0.60$
- Standard deviation of first asset = 0.0357
- Standard deviation of second asset = 0.0424
- Covariance between the two assets = 0.0015

$$\text{Variance of the portfolio} = 0.4^2 * 0.0357^2 + 0.6^2 * 0.0424^2 + 2 * 0.4 * 0.6 * 0.0015 = 0.00157$$

$$\text{Standard deviation of the portfolio} = \sqrt{0.00157} = 0.0396$$

4. Topics in Probability

4.1. Bayes' Formula

Bayes' formula is a rational method for updating or adjusting the probability of an event based on new information. According to Bayes' formula, the updated probability of an event given new information is:

$$P(\text{Event} | \text{Information}) = \frac{P(\text{Information} | \text{Event})}{P(\text{Information})} * P(\text{Event})$$

Example

Consider a factory that has three assembly lines. The percentage of output produced at each assembly line is as follows: Line A = 45%, Line B = 35%, Line C = 20%. The output defective from each line is estimated to be 3%, 5%, and 4%, respectively. Given that the product is defective, what is the probability that it came from Line C?

Solution:

When dealing with questions related to Bayes' formula, the first step is to reproduce the information in probability notation:

$$P(\text{Line A}) = 0.45; P(\text{Not Line A}) = 0.55$$

$$P(\text{Line B}) = 0.35; P(\text{Not Line B}) = 0.65$$

$$P(\text{Line C}) = 0.20; P(\text{Not Line C}) = 0.80$$

$$P(\text{Defective} \mid \text{Line A}) = 0.03, P(\text{Defective} \mid \text{Line B}) = 0.05, P(\text{Defective} \mid \text{Line C}) = 0.04$$

$$P(\text{Defective}) = 0.45 \times 0.03 + 0.35 \times 0.05 + 0.20 \times 0.04 = 0.039$$

Next write down the Bayes formula:

$$P(\text{Event} \mid \text{Information}) = \frac{P(\text{Information} \mid \text{Event})}{P(\text{Information})} \times P(\text{Event})$$

We then have to distinguish between the event and the information and plug the relevant values into the formula. In this case, the information is that the product is defective. Hence, the formula can be written as:

$$P(\text{Line C} \mid \text{Defective}) = \frac{[P(\text{Defective} \mid \text{Line C}) * P(\text{Line C})]}{P(\text{Defective})} = \frac{0.04 * 0.20}{0.039} = 20.51\%$$

4.2. Principles of Counting

In counting, enumeration (counting the outcomes one by one) is the most basic resource. This process is difficult and is prone to error. We will discuss shortcuts and principles of counting, which make the process easier.

Multiplication Rule of Counting

The first of these principles is the multiplication rule. It states that ‘if one task can be done in n_1 ways, and a second task, given the first, can be done in n_2 ways, and a third task, given the first two tasks, can be done in n_3 ways and so on for k tasks, then the number of ways the k tasks can be done is $(n_1) (n_2) (n_3) \dots (n_k)$. So the multiplication rule for counting can be expressed as:

$$\text{Number of ways of doing } k \text{ tasks} = n_1 \times n_2 \times n_3 \dots n_k$$

where ,

n_1 = number of ways of doing the first task,

n_2 = number of ways of doing the second task and so on

Example

Consider a simple example. Suppose we have three steps in an investment decision process. The first step can be done in 2 ways, the second step can be done in 4 ways and the third in 3 ways. In how many ways can the investment decision be made?

Solution:

Following the multiplication rule, there are $(2) (4) (3) = 24$ ways of making the investment decision.

Notice that there are three groupings in this problem. From each group, only one step can be selected.

Factorial

Another counting principle relates to the assignment of members of a group to an equal number of positions. The number of ways we can assign every member of a group of size n to n slots is $n!$ (read as **n factorial**) $= n (n - 1) (n - 2) (n - 3) \dots 1$. By convention, $0! = 1$. The difference between the multiplication rule and factorial is that there is only one group in a factorial. It involves arranging the set of items within the group and the order in which the items are arranged matters. The formula is:

Number of ways of assigning group of size n to n tasks $= n!$

Example

There are five equity analysts covering five emerging countries. In how many ways can the countries be assigned to the analysts?

Solution:

The total number of ways the assignments can be made $= 5! = 120$

Labeling

Labeling refers to the number of ways that n items can be labelled with k different labels with n_1 of the first type, n_2 of the second type, and so on. This can be expressed as:

Number of ways in which n items can be labelled using k labels $= \frac{n!}{[(n_1!)(n_2!)\dots(n_k!)]}$

Example

A portfolio consists of eight stocks. The goal is to designate four of the stocks as "long-term holds," three of the stocks as "short-term holds," and one stock a "sell." How many ways can these labels be assigned to the eight stocks?

Solution:

Notice that there are eight items (stocks) that are to be labelled in three different ways.

$$\frac{8!}{4! * 3! * 1!} = 280$$

Combination

A special case of the labelling is the combination formula. It is the number of ways to choose r objects from a total of n objects, when the order in which the r objects are listed does not matter. This can be expressed as:

$${}_nC_r = n! / [(n - r)! r!]$$

where:

n = number of objects

r = number of objects chosen from n objects

${}_nC_r$ = number of ways to choose r objects from n objects where order does not matter

Example

A portfolio manager wants to eliminate four stocks from a portfolio that consists of six stocks. How many ways can the four stocks be sold when the order of the sale is NOT important?

Solution:

Using the formula for combination, we get the number of ways the four stocks can be sold
 $= 6! / [(6 - 4)! 4!] = 15$.

Permutation

Permutation is the number of ways to choose r objects from a total of n objects, when the order in which the r objects are listed does matter. It is expressed as:

$${}_nP_r = \frac{n!}{(n - r)!}$$

where:

n = number of objects

r = number of objects chosen from n objects

Example

Assume that in a portfolio of eight stocks, we decide to sell three stocks. How many ways can we choose three of the eight to sell if the order of sale does matter?

Solution:

Using the formula for permutation, we can find the number of ways to sell three of the eight stocks where order matters: $\frac{8!}{(8 - 3)!} = 336$.

Instructor's Note: given a problem, use the following pointers to identify the correct counting method to apply.

- Factorial: if there is **one** group of size n and n items/tasks to be assigned, number of ways = $n!$
- Labelling: used when there are three or more labels. Each item/member of a group must be applied a label.
- Combination: used when there are two groups of a certain size, say n and r . Use combination when the order of choosing r objects from n objects does **NOT** matter.
- Permutation: used when there are two groups of a certain size, say n and r . Use permutation when the order of choosing r objects from n objects does matter.
- Combination and permutation functions are available on the financial calculator and should be used rather than the formula.

Summary

LO.a: Define a random variable, an outcome, an event, mutually exclusive events, and exhaustive events.

- A random variable is an uncertain quantity/number.
- An outcome is the observed value of a random variable.
- An event can be a single outcome or a set of outcomes.
- Mutually exclusive events are events that cannot happen at the same time.
- Exhaustive events are those that include all possible outcomes.

LO.b: State the two defining properties of probability and distinguish among empirical, subjective, and a priori probabilities.

The two defining properties of a probability are:

- The probability of any event E is a number between 0 and 1: $0 \leq P(E) \leq 1$.
- The sum of the probabilities of any set of mutually exclusive and exhaustive events equals 1.

The methods of estimating probabilities are:

- Empirical probability: Based on analyzing the frequency of an event's occurrence in the past.
- A priori probability: Based on formal reasoning and inspection rather than personal judgment.
- Subjective probability: Informed guess based on personal judgment.

LO.c: State the probability of an event in terms of odds for and against the event.

Odds for E = $P(E) / [1 - P(E)]$.

Odds against E = $[1 - P(E)] / P(E)$.

LO.d: Distinguish between unconditional and conditional probabilities.

Unconditional probability (marginal probability) is the probability of an event occurring irrespective of the occurrence of other events. It is denoted as $P(A)$.

Conditional probability is the probability of an event occurring given that another event has occurred. It is denoted as $P(A|B)$, which is the probability of event A given that event B has occurred.

LO.e: Explain the multiplication, addition, and total probability rules.

Multiplication rule is used to determine the joint probability of two events. It is expressed as:

$$P(AB) = P(A|B) P(B)$$

Addition rule is used to determine the probability that at least one of the events will occur. It is expressed as:

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

The total probability rule is used to calculate the unconditional probability of an event, given conditional probabilities. It is expressed as:

$$P(A) = P(A|S_1) P(S_1) + P(A|S_2) P(S_2) + \dots + P(A|S_n) P(S_n)$$

LO.f: Calculate and interpret 1) the joint probability of two events, 2) the probability that at least one of two events will occur, given the probability of each and the joint probability of the two events, and 3) a joint probability of any number of independent events.

Consider the example that you have two stocks A and B. The probability that stock A will pay out a dividend this quarter is 60% and the probability that stock B will pay out a dividend this quarter is 50%. Assume that the two events are independent. The joint probability of A and B i.e. both A and B issue a dividend this quarter is $0.6 * 0.5 = 0.3$.

The probability that at least one of two events, A or B, will occur is $P(A \text{ or } B) = P(A) + P(B) - P(AB) = 0.6 + 0.5 - 0.6 * 0.5 = 0.8$.

LO.g: Distinguish between dependent and independent events.

If the occurrence of one event does not influence the occurrence of the other event, then the events are called independent events.

$$\text{i.e. } P(A|B) = P(A) \text{ or } P(B|A) = P(B)$$

If the probability of an event is affected by the occurrence of another event then it is called a dependent event.

LO.h: Calculate and interpret an unconditional probability using the total probability rule.

Using the total probability rule the unconditional probability of A can be computed as:

$$P(A) = P(A|S_1) P(S_1) + P(A|S_2) P(S_2) + \dots + P(A|S_n) P(S_n)$$

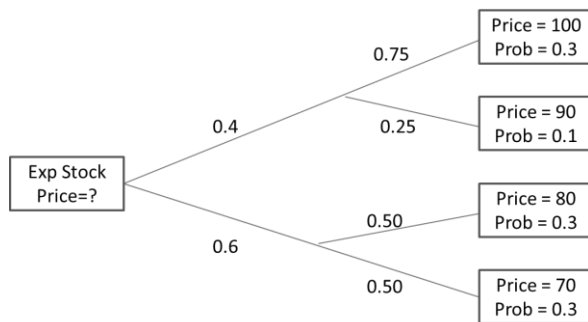
Where $S_1, S_2 \dots S_n$ are mutually exclusive and exhaustive events.

LO.i: Explain the use of conditional expectation in investment applications.

The expected value is heavily used in investment applications e.g. forecasting EPS of a company, estimating rewards of alternative investments etc. When you refine your expectations or forecasts, you are typically making adjustments based on new information or events; this is actually the use of conditional expected values.

LO.j: Explain the use of a tree diagram to represent an investment problem.

The tree diagram in an investment problem can help plot the probabilities of various outcomes and depict expected values based on the paths chosen and selection made at each node. For example:

**LO.k: Calculate and interpret covariance and correlation.**

Covariance is a measure of how two variables move together. Given two random variables R_i and R_j , the covariance between R_i and R_j is given by:

$$\text{Cov}(R_i, R_j) = E[(R_i - ER_i)(R_j - ER_j)]$$

Where,

ER_i = expected return for variable R_i

ER_j = expected return for variable R_j

Correlation is a standardized measure of the linear relationship between two variables with values ranging between -1 and +1. The formula is given by:

$$\rho(A, B) = \text{Cov}(A, B) / \sigma(A) \sigma(B)$$

LO.l: Calculate and interpret the expected value, variance, and standard deviation of a random variable and of returns on a portfolio.

The formula for calculating the expected portfolio return is:

$$E(R_P) = w_1 E(R_1) + w_2 E(R_2) + \dots + w_n E(R_n)$$

Variance can be computed as:

$$\sigma^2(R_P) = w_1^2 \sigma_1^2(R_1) + w_2^2 \sigma_2^2(R_2) + 2w_1 w_2 \text{Cov}(R_1, R_2)$$

LO.m: Calculate and interpret covariance given a joint probability function.

Given two random variables R_i and R_j , the covariance between R_i and R_j is given by:

$$\text{Cov}(R_i, R_j) = E[(R_i - ER_i)(R_j - ER_j)]$$

where:

ER_i = expected return for variable R_i

ER_j = expected return for variable R_j

LO.n: Calculate and interpret an updated probability using Bayes' formula.

Bayes' formula is a rational method for updating or adjusting the probability of an event based on new information. According to Bayes' formula, the updated probability of an event given new information is:

$$P(\text{Event} \mid \text{Information}) = \frac{P(\text{Information} \mid \text{Event})}{P(\text{Information})} * P(\text{Event})$$

LO.o: Identify the most appropriate method to solve a particular counting problem, and solve counting problems using factorial, combination, and permutation concepts.

The number of ways we can assign every member of a group of size n to n slots is $n!$

Number of ways in which n items can be labelled using k labels = $\frac{n!}{[(n_1!)(n_2!)...(n_k!)]}$

The combination formula gives the number of ways to choose r objects from a total of n objects, when the order in which the r objects are listed does not matter.

$${}^nC_r = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

The permutation formula gives the number of ways to choose r objects from a total of n objects, when the order in which the r objects are listed does matter.

$${}^nP_r = \frac{n!}{(n-r)!}$$

Practice Questions

1. If events A and B are mutually exclusive, then which of the following is *true*?
 - A. $P(A|B) = P(A)$.
 - B. $P(AB) = P(A) \times P(B)$.
 - C. $P(A \text{ or } B) = P(A) + P(B)$.
 2. Which of the following values can be the probability of an event?
 - A. -0.23.
 - B. 0.68.
 - C. 1.36.
 3. Which of the following is *most likely* an empirical probability?
 - A. The probability of a particular outcome when exactly 6 equally likely possible outcomes exist.
 - B. The probability that US stock returns exceed long-term corporate bond returns over a 5 year period, based on Zeller Financial data.
 - C. An updated probability of an event arrived at using Bayes' formula and the perceived prior probability of the event.
 4. The probability that the stock market will increase tomorrow is $3/5$. The probability of an increase in the stock market stated as 'odds for' is:
 - A. three-to-two.
 - B. two-to-three.
 - C. three-to-five.
 5. A fund manager has noted that during the past five years 60 percent of the stocks in his portfolio have paid a cash dividend and 20 percent of the stocks have paid a stock dividend. If 65 percent of the stocks have paid a dividend of any kind, the joint probability of a stock paying both a cash dividend and a stock dividend is *closest* to:
 - A. 10%.
 - B. 15%.
 - C. 20%.
 6. Two events are said to be independent if the occurrence of one event:
 - A. implies that the second event cannot occur.
 - B. implies that the second event is certain to occur.
 - C. does not affect the probability of the occurrence of the other event.
 7. The correlation of returns between Stocks A and B is 0.60. The covariance between these two securities is 0.0056 and the standard deviation of the returns of Stock B is 18%. The variance of returns for Stock A is:
-

- A. 0.0027.
- B. 0.0035.
- C. 0.0048.

8. The following information on two portfolios is given:

	Fund A	Fund B
Portfolio weights (%)	40	60
Expected returns (%)	15	10
Standard deviations (%)	20	16
Correlation between the returns of Fund A and Fund B	0.5	

The expected return and the standard deviation of the portfolio are respectively *closest* to:

- A. 12.5% and 22.3%.
 - B. 12% and 15.2%.
 - C. 13% and 16.5%.
9. The joint probabilities for X and Y are $P(X=10, Y=3) = 0.4$, and $P(X=20, Y=6) = 0.6$, then the Covariance of XY is *closest* to:
- A. 7.2
 - B. 6.5.
 - C. 6.6.
10. A researcher is studying the link between exchange rate movements and the discount rate set by the country's bank. He uses historical data to determine that the probability of exchange rate rising or falling over the next month is 60% and 30% respectively while the probability that the exchange rate stays the same is 10%. Some days later, he receives information that the central bank will increase the discount rate. The researcher comes up with the following subjective probabilities that the central bank will increase the discount rate given the scenarios that exchange rate rises, falls or stays the same:
- $P(\text{increased discount rate} | \text{exchange rate increases}) = 65\%$
 - $P(\text{increased discount rate} | \text{exchange rate stays same}) = 15\%$
 - $P(\text{increased discount rate} | \text{exchange rate decreases}) = 20\%$
- What is the probability that the exchange rate will fall given the new information that the central bank will increase the discount rate?
- A. 18.79%.
 - B. 15.65%.
 - C. 12.90%
11. An investor wants to purchase 7 stocks for his portfolio from an universe of 10 stocks. If the order in which he buys the stocks is not important, then how many potential 7 stock

combinations do we have?

- A. 120
- B. 140
- C. 160.

12. A fund manager wants to sell 3 bonds from his portfolio of 8 bonds. If the order in which he sells the bonds is important, then how many potential 3-bond combinations do we have?

- A. 56.
- B. 336.
- C. 512.

Solutions

1. C is correct. Mutually exclusive events are those events which cannot happen together, i.e. there is no intersection between the two events. Therefore, both $P(A|B)$ and $P(AB)$ must be equal to zero.
2. B is correct. The probability of an event has to be between 0 and 1.
3. B is correct. This is an example of empirical probability since it is based on past data. Option A is A priori probability since it is based on logical reasoning. Option C is subjective probability since it is based on personal judgment.
4. A is correct.
Odds for E = $P(E) / [1 - P(E)] = (3/5) / (2/5) = 3/2 = \text{three to two}$.
5. B is correct. The probability that at least one of two events will occur is the sum of the probabilities of the separate events less the joint probability of the two events. $P(A \text{ or } B) = P(A) + P(B) - P(AB)$
 $65\% = 60\% + 20\% - P(AB)$; therefore $P(AB) = 15\%$
6. C is correct. Two events are said to be independent if the occurrence of one event does not affect the probability of the occurrence of the other event.
7. A is correct.
$$\text{Corr}(A, B) = \frac{\text{Cov}(A, B)}{\sigma(A)\sigma(B)}$$
$$0.6 = \frac{0.0056}{\sigma(A)0.18}$$
Therefore $\sigma(A) = 0.0518$. The variance of returns for stock A = $0.0518^2 = 0.0027$
8. B is correct.
The expected return on the portfolio is $0.4 \times 15\% + 0.6 \times 10\% = 12\%$
Calculate the portfolio standard deviation of returns as follows:
 $\sqrt{(.40^2 \times .20^2 + .60^2 \times .16^2 + 2 \times .40 \times .60 \times .5 \times .20 \times .16)} = 0.152$ and 15.2%
9. A is correct. The joint probabilities in the question are given as follows:

X, Y	3	6
10	0.4	
20		0.6

$$E[X] = 0.4(10) + 0.6(20) = 16$$

$$E[Y] = 0.4(3) + 0.6(6) = 4.8$$

$$\text{Cov}[XY] = 0.4(10-16)(3-4.8) + 0.6(20-16)(6-4.8) = 7.2$$

10. C is correct. According to Bayes' Theorem:

Updated probability of event given the new information

= (Probability of new information given event ÷ unconditional probability of new information) x prior probability of event

In order to proceed with the given data, we need to calculate the unconditional probability of new information i.e. the probability of an increase in the discount rate.

P (increased discount rate)

= P (increased discount rate| exchange rate increases) x P (exchange rate increases) + P (increased discount rate| exchange rate stays same) x P (exchange rate stays same) + P (increased discount rate| exchange rate decreases) x P(exchange rate decreases)

= (0.65 x 0.60) + (0.15 x 0.1) + (0.2 x 0.3)

= 0.465

= 46.5%

$$P(E/I) = P(E) \times \frac{P(I/E)}{P(I)}$$

Using the unconditional probability and Bayes' Theorem, we can calculate updated probability of event given the new information about discount rates as:

P (exchange rate decreases| increased discount rate)

== [P (increased discount rate| exchange rate decreases) ÷ P (increased discount rate)] x P (exchange rate decreases)

= (0.20 ÷ 0.465) x 0.30

= 12.90

11. A is correct. Since the order is not important we will use combination. ${}_{10}C_7 = 120$

12. B is correct. Since the order is important we will use permutation. ${}_8P_3 = 336$.

R10 Common Probability Distributions

1. Introduction

In nearly all investment decisions, we work with random variables. To make probability statements about a random variable, we need to understand its probability distribution. A probability distribution specifies the probabilities of all possible outcomes of a random variable. In this reading, we will look at the following four probability distributions:

- Uniform
- Binomial
- Normal
- Lognormal

2. Discrete Random Variables

A random variable is a variable whose outcome cannot be predicted. There are two basic types of random variables:

- A **discrete random variable** is one where we can list all the possible outcomes. For example, the stocks traded on the New York Stock Exchange are quoted in ticks of \$0.01. Quoted stock price is therefore a discrete random variable.
- A **continuous random variable** is where the number of points between the lower and upper bounds are essentially infinite. Rate of return is an example of a continuous random variable.

Probability distribution specifies the probabilities of all the possible outcomes for a random variable.

- Probability function specifies probability that random variable takes on a specific value.
- Probability density function is used for continuous random variables.

2.1. The Discrete Uniform Distribution

The discrete uniform distribution has a finite number of specified outcomes and each outcome is equally likely. Consider a roll of a dice. The outcome is a random variable and it can take a value of 1 to 6. The probability that the random variable takes on any of these values is the same for all outcomes. With six outcomes, $p(x) = 1/6$ for all values of X ($X = 1, 2, 3, 4, 5, 6$). The table below summarizes the two views of this random variable – the probability function and the cumulative distribution function.

$X = x$	Probability Function $p(x) = P(X = x)$	Cumulative Distribution Function $F(x) = P(X \leq x)$
1	1/6	1/6
2	1/6	2/6
3	1/6	3/6

4	1/6	4/6
5	1/6	5/6
6	1/6	6/6

Example

Using the table above, find the following probabilities:

4. $F(4)$
5. $P(3 \leq X < 6)$
6. $F(9)$

Solution to 1:

To find $F(4)$, we must find the cumulative probability of $P(X \leq 4)$ using the cumulative distribution function (third column). From the table, we can see that $P(X \leq 4) = 4/6 = 2/3$. Therefore, the probability is $2/3$.

Solution to 2:

To find $P(3 \leq X < 6)$, we need to find the sum of three probabilities: $p(3)$, $p(4)$, $p(5)$. This is $1/6 + 1/6 + 1/6 = 3/6 = 1/2$.

Solution to 3:

To find $F(9)$, we must find the cumulative probability of $P(X \leq 9)$. This includes all possible outcomes, hence the probability is 1.

2.2. The Binomial Distribution

The building block of the binomial distribution is the **Bernoulli random variable**. Suppose we have a trial such as flipping a coin that produces one of two outcomes. Such a trial is a **Bernoulli trial**. We define one outcome as a 'success' and the other outcome as a 'failure'. This can be expressed as:

$$P(Y = 1) = p$$

$$P(Y = 0) = 1 - p$$

where:

p = probability that the trial is a success

A **binomial random variable**, X , is the number of successes in a given number (n) of Bernoulli trials. Continuing with the coin example, say we flip the coin 10 times and we define success as 'Heads'. Clearly with 10 flips we can get 0 to 10 successes.

The probability distribution of a binomial random variable for the probability of " x " success in " n " trials is calculated using the following formula:

$$P(x) = P(X = x) = {}_n C_x p^x (1 - p)^{n - x}$$

where:

p = the probability of success on each trial
 x = number of successes
 n = number of trials

Instructor's Note:

Two important points help illustrate the intuition behind the formula:

- The successes can be in any order. That is why we use the combination function and not the permutation function.
- The events are independent. That is why we simply multiply the probability of each event.

Example

If we flip a fair coin ($p = 0.5$) ten times ($n = 10$), what is the probability of seven successes?

Solution:

$$P(7) = P(X = 7) = {}_{10}C_7 0.5^7 0.5^3 = 0.117$$

Mean and variance of a binomial variable

The mean and variance of a binomial variable can be calculated as:

Random Variable	Mean	Variance
Bernoulli, $B(1, p)$	p	$p(1 - p)$
Binomial, $B(n, p)$	np	$np(1 - p)$

For our coin-flip example, the mean value of the binomial random variable is $np = 10 \times 0.5 = 5$. The intuition: if we perform the binomial experiment several times, where each experiment refers to 10 coin-flips, on average we will have 5 successes. The actual number of successes will be distributed equally on either side of the mean value. The random value for every trial moves closer to the expected value as the number of trials grows.

Example

Over the last 10 years, Abro corporation's EPS increased year over year six times and decreased year over year four times. You decide to model the number of EPS increases for the next decade as a binomial random variable.

7. If success is defined as an increase in the annual EPS, determine the probability of success.
8. What is the probability that EPS will increase in exactly 5 of the next 10 years?
9. Calculate the expected number of yearly EPS increases during the next 10 years.
10. Calculate the variance and standard deviation of the number of yearly EPS increases during the next 10 years.

Solution to 1:

There are only two possible outcomes: increase in the EPS and no increase in the EPS.

Probability of success: $p = 6/10 = 0.6$

Probability of failure: $1 - p = 1 - 0.6 = 0.4$

Solution to 2:

Using the binomial model:

$$P(X = 5) = {}_n C_x p^x q^{n-x}$$

$$P(X = 5) = {}_{10} C_5 0.6^5 0.4^5$$

$$P(X = 5) = 252 * 0.07776 * 0.01024 = 20.06\%$$

Solution to 3:

Expected number of yearly EPS increases: $E(x) = np = 10 * 0.6 = 6$

Solution to 4:

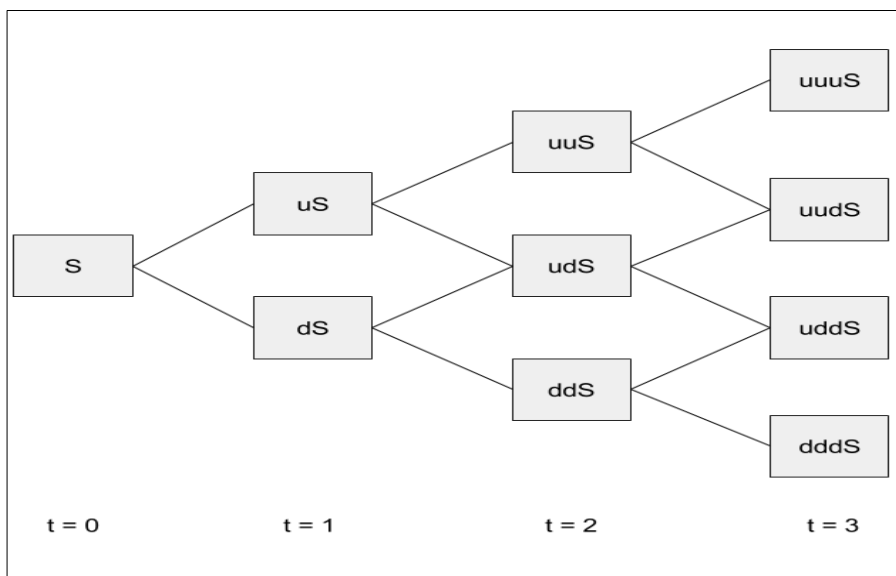
$$\text{Variance} = np(1 - p) = 6 * 0.4 = 2.4$$

$$\text{Standard Deviation} = \sqrt{2.4} = 1.549$$

The variance of the distribution is calculated as $np(1 - p) = 10 * 0.5 * (1 - 0.5) = 2.5$

Binomial tree

A binomial tree can be used to model stock price movements. Refer to the tree diagram below. 'S' represents the initial stock price. 'u' represents an up move and 'd' represents a down move. The nodes show each possible value of the stock after 1, 2 and 3 time periods.



Example

Consider an initial stock price of \$100. In one time period, the stock can either rise by a factor of 1.1 or go down by a factor of 1/1.1. In any given time period, the probability of an up move is 0.6 and the probability of a down move is 0.4. After two periods, what are the possible stock prices and their respective probabilities? What is the expected stock price?

Solution:

uuS = $1.1 \times 1.1 \times 100 = 121$ with probability $0.6 \times 0.6 = 0.36$

udS = $1.1 \times 1/1.1 \times 100 = 100$ with probability $0.6 \times 0.4 = 0.24$

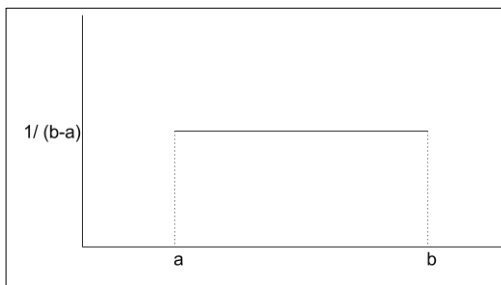
duS = $1/1.1 \times 1.1 \times 100 = 100$ with probability $0.4 \times 0.6 = 0.24$

ddS = $1/1.1 \times 1/1.1 \times 100 = 82.64$ with probability $0.4 \times 0.4 = 0.16$

Expected stock price = $121 \times 0.36 + 100 \times 0.24 + 100 \times 0.24 + 82.64 \times 0.16 = \104.78

3. Continuous Random Variables**3.1. Continuous Uniform Distribution**

The continuous uniform distribution is defined over a range from a lower limit 'a' to an upper limit 'b'. These limits serve as the parameters of the distribution.



The probability that the random variable will take a value between x_1 and x_2 , where x_1 and x_2 both lie within the range is given by:

$$P(x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b - a}$$

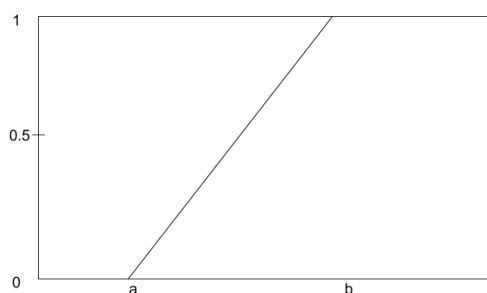
Example

X is a uniformly distributed continuous random variable between 10 and 20. Calculate the probability that X will fall between 12 and 18.

Solution:

$$P(12 \leq X \leq 18) = \frac{18 - 12}{20 - 10} = 0.6$$

The cumulative distribution function for a continuous random variable is shown below:



Example

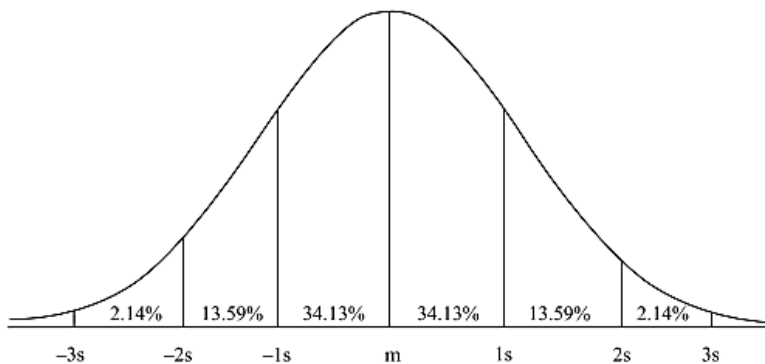
A commodity analyst predicts that the price per ounce of gold three years from now will be between \$1,500 and \$1,700. Assume gold prices follow a continuous uniform distribution. What is the probability that the price will be less than \$1,600 three years from now?

Solution:

$F(1,600) = \frac{1,600 - 1,500}{1,700 - 1,500} = 50\%$ The probability that gold price will be less than \$1,600 per ounce three years from now is 50%.

3.2. The Normal Distribution

The normal distribution is the most extensively used probability distribution in quantitative work. A normal distribution is symmetrical and bell-shaped as shown in the graph below:



In this figure 'm' stands for mean, 1s means one standard deviation, 2s means two standard deviations, and so on. We can make the following probability statements for a normal distribution:

- Approximately 68% of all observations fall in the interval $m \pm s$.
- Approximately 95% of all observations fall in the interval $m \pm 2s$.
- Approximately 99% of all observations fall in the interval $m \pm 3s$.

The intervals indicated above are easy to remember but are only approximate for the stated probabilities. More precise intervals (confidence intervals) are:

- 90% of all observations are in the interval $m \pm 1.65s$.
- 95% of all observations are in the interval $m \pm 1.96s$.
- 99% of all observations are in the interval $m \pm 2.58s$.

The characteristics of a normal distribution are as follows:

- The normal distribution is completely described by two parameters – its mean, μ , and variance, σ^2 . We indicate this as $X \sim N(\mu, \sigma^2)$.
- The normal distribution has a skewness of 0 (it is symmetric) and a kurtosis (measure of peakedness) of 3. Due to the symmetry, the mean, median and mode are all equal for a normal random variable.

- A linear combination of two or more normal random variables is also normally distributed.

Standard normal distribution

The normal distribution with mean (μ) = 0 and standard deviation (σ) = 1 is called the standard normal distribution.

The formula for standardizing a random variable X is:

$$Z = \frac{(X - \mu)}{\sigma}$$

where:

μ is the population mean.

σ is the population standard deviation.

The Z-table is used to find the probability that X will be less than or equal to a given value. Suppose we have a normal random variable, X, with $\mu = 10$ and $\sigma = 2$. If the value of X is 11, we standardize X with $Z = (11 - 10)/2 = 0.5$.

The probability that we will observe a value less than 11 for $X \sim N(10, 2)$ is exactly the same as the probability that we will observe a value less than 0.5 for $Z \sim N(0, 1)$.

We can answer probability questions about X using standardized values and probability tables for Z. A snapshot of a table showing cumulative probabilities for a standard normal distribution is shown below.

x or z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224

To find the probability that a standard normal variable is less than or equal to 0.5, for example, locate the row that contains 0.50, look at the 0 column, and find the entry 0.6915. Thus, $P(Z \leq 0.5) = 0.6915$ or 69.15%.

The probability that a standard normal variable is less than or equal to 0 is 0.5000. This is true by definition because the mean of a standard normal distribution is 0. The table above validates this fact.

For a non-negative number x, we can use $N(x)$ directly from the table. For a negative number $-x$, $N(-x) = 1.0 - N(x)$. Essentially, we are using the fact that the normal distribution is symmetric around the mean.

Example

A portfolio has a mean return of 15% and a standard deviation of return of 20% per year. What is the probability that the portfolio return would be below 18%? We are given the following information from the z-table: $P(Z < 0.15) = 0.5596$, $P(Z > 0.15) = 0.4404$, $P(Z < 0.18) = 0.5714$, $P(Z > 0.18) = 0.4286$.

Solution:

$$P\left(Z < \frac{X - \mu}{\sigma}\right) = P\left(Z < \frac{0.18 - 0.15}{0.20}\right) = P(Z < 0.15) = 0.5596$$

Univariate v/s multivariate distributionUnivariate distribution

A univariate distribution describes the probability distribution of a single random variable. For example, the distribution of expected return of one stock from a portfolio.

Multivariate distribution

A multivariate distribution describes the probability distribution for a group of related random variables. For example, the distribution of expected return of a portfolio with multiple stocks.

A multivariate normal distribution for the returns on n stocks is completely defined by the following three sets of parameters:

- Mean returns on the individual stocks (n means in total).
- Variances of the individual stocks (n variances in total).
- Pairwise return correlations between the stocks ($n(n - 1)/2$ distinct correlations in total).

3.3. Applications of the Normal Distribution**Shortfall risk**

Shortfall risk is the risk that portfolio's return will fall below a specified minimum level of return over a given period of time.

Safety first ratio

Safety first ratio is used to measure shortfall risk. It is calculated as:

$$\text{SF-Ratio} = \frac{[R_P - R_L]}{\sigma_P}$$

where:

R_P = Expected portfolio return

R_L = Threshold level

σ_P = Standard deviation of portfolio returns

A portfolio with a higher safety first ratio is preferred over a portfolio with a lower safety

first ratio.

Example

An investor is considering two portfolios A and B. Portfolio A has an expected return of 10% and a standard deviation of 2%. Portfolio B has an expected return of 15% and a standard deviation of 10%. The minimum acceptable return for the investor is 8%. According to Roy's safety first criteria, which portfolio should the investor select?

Solution:

$$SF_A = \frac{10 - 8}{2} = 1$$

$$SF_B = \frac{15 - 8}{10} = 0.7$$

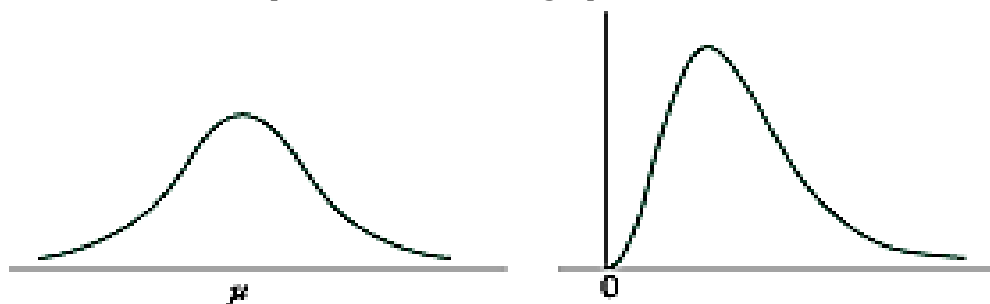
Since A has a higher safety first ratio, the investor should select portfolio A.

Roy's safety first criteria

It states that an optimal portfolio minimizes the probability that the actual portfolio return will fall below the target return.

3.4. The Lognormal Distribution

If x is a random variable that is normally distributed, then to create a lognormal distribution of x we take e^x and plot the values on a graph.



Normal Distribution

Lognormal Distribution

The properties of a lognormal distribution are:

- It cannot be negative.
- The upper end of its range extends to infinity.
- It is positively skewed.

A lognormal distribution is often used to model asset prices because the asset prices need to be positive, they cannot be negative.

Discretely versus continuously compounded rates of return

Discretely compounded rates of returns have defined compounding periods such as quarterly, monthly etc. As we decrease the length of the compounding period, the effective

annual rate rises. For continuous compounding, the EAR is given by:

$$\text{EAR} = e^r - 1$$

If we are given the holding period return over any time period, we can calculate the equivalent continuously compounded rate of return for that period as:

$$r = \ln(\text{HPR} + 1)$$

Example

If the holding period return of a stock was 10% for a period of one year. What is the equivalent continuously compounded rate of return for the year?

Solution:

$$r = \ln(0.1 + 1) = 0.0953 = 9.53\%$$

4. Monte Carlo Simulation

Monte Carlo simulation is a computer simulation used to simulate possible security prices based on risk factors. As input, it uses randomly generated values for risk factors based on their assumed distributions. It processes this information as per the specified model and runs thousands of iterations. Then it gives the distribution of the expected value of the security as output.

Major applications include:

- financial planning
- developing VAR estimates
- valuing complex securities

Limitations include:

- It is fairly complex and will provide answers that are no better than the assumptions.
- Simulation is not an analytical method but a statistical one.

Monte Carlo simulation versus historical simulation

The historical simulation uses actual past distribution of risk factors as input. Whereas, Monte Carlo simulation uses randomly generated values of the risk factors based on their assumed distributions.

The limitations of historical simulation are:

- It cannot take into account the effect of significant events that did not occur during the sample period.
- It cannot perform a 'what if' analysis when the 'if' scenario has not happened in the past.

Summary

LO.a: Define a probability distribution and distinguish between discrete and continuous random variables and their probability functions.

A random variable is a variable whose outcome cannot be predicted. A probability distribution lists all possible outcomes of a random variable along with their associated probabilities.

A discrete random variable is one for which the number of possible outcomes can be counted. It has measurable probabilities associated with each specific outcome.

A continuous random variable is one for which we cannot count the number of possible outcomes. Therefore, probabilities cannot be associated with specific outcomes, instead, it has to be assigned to a particular range.

LO.b: Describe the set of possible outcomes of a specified discrete random variable.

The set of possible outcomes of a discrete random variable is finite.

- $p(x) = 0$ means that x cannot occur.
- $p(x) > 0$ means that x can occur.
- $p(x) = 1$ means that x is the only possible outcome.

LO.c: Interpret a cumulative distribution function.

The cumulative distribution function gives the probability that the random variable will be less than or equal to a specific value.

It is denoted by $F(x) = P(X \leq x)$.

LO.d: Calculate and interpret probabilities for a random variable, given its cumulative distribution function.

The probability that an outcome will be less than or equal to a specific value is represented by the area under the cumulative probability distribution to the left of that value.

LO.e: Define a discrete uniform random variable, a Bernoulli random variable, and a binomial random variable.

A discrete uniform random variable is one where the probability of all the possible outcomes is equal. For example, the roll of a dice.

A Bernoulli trial is an experiment that has only two possible outcomes: a success or a failure. For example, the toss of a coin.

If the experiment is carried out n times, the number of success (denoted by X) is called a Bernoulli random variable.

The distribution that X follows is known as the binomial distribution.

LO.f: Calculate and interpret probabilities given the discrete uniform and the binomial

distribution functions.

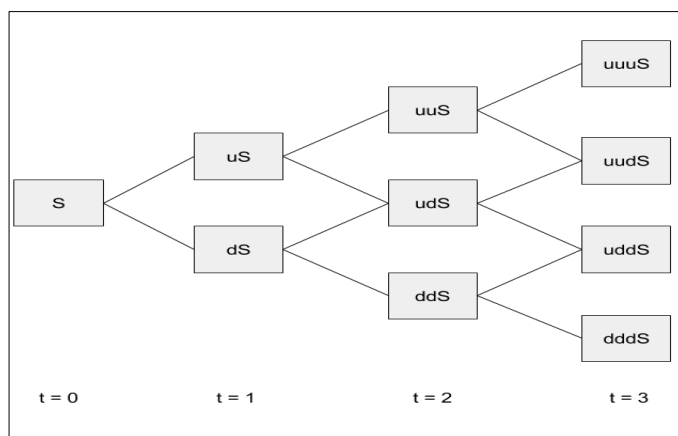
Probabilities for a discrete uniform distribution: If the total number of outcomes is n , then the probability of each outcome = $1/n$.

Probabilities for a uniform random variable: The probability distribution of a binomial random variable for the probability of x successes in n trials is calculated using the following formula:

$$P(x) = P(X = x) = {}_n C_x p^x (1 - p)^{n-x}$$

LO.g: Construct a binomial tree to describe stock price movement.

A binomial tree can be used to model stock price movements. 'S' represents the initial stock price. 'u' represents an up move and 'd' represents a down move. The nodes show each possible value of the stock after 1, 2 and 3 time periods.

**LO.h: Define the continuous uniform distribution and calculate and interpret probabilities, given a continuous uniform distribution.**

The continuous uniform distribution is defined over a range from a lower limit 'a' to an upper limit 'b'. The probability that the random variable will take a value between x_1 and x_2 , where x_1 and x_2 both lie within the range is given by:

$$P(x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b - a}$$

LO.i: Explain the key properties of the normal distribution.

A normal distribution is a bell-shaped curve, with two identical halves.

It is completely described by two parameters its mean (μ) and its variance (σ^2). This is stated as $X \sim N(\mu, \sigma^2)$.

It has a skewness of 0 and a kurtosis of 3.

A linear combination of two or more random variables is also normally distributed.

LO.j: Distinguish between a univariate and a multivariate distribution, and explain the

role of correlation in the multivariate normal distribution.

A univariate distribution describes a single random variable. An example is the expected return on a particular stock. A multivariate distribution specifies the probabilities for a group of related random variables. An example is the expected return on a portfolio of multiple stocks.

A multivariate normal distribution for the returns on n stocks is completely defined by three lists of parameters:

- The list of the mean returns on the individual securities (n means in total).
- The list of the securities' variances of return (n variances in total).
- The list of all the distinct pairwise return correlations ($n(n-1)/2$ distinct correlations in total).

LO.k: Determine the probability that a normally distributed random variable lies inside a given interval.

The property of normal distribution gives us intervals stated below:

- 90% of all observations are in the interval $m \pm 1.65s$.
- 95% of all observations are in the interval $m \pm 1.96s$.
- 99% of all observations are in the interval $m \pm 2.58s$.

This implies that if a random variable is less than or equal to $m \pm 1.65s$, then the probability of it lying inside the distribution is 90%.

LO.l: Define the standard normal distribution, explain how to standardize a random variable, and calculate and interpret probabilities using the standard normal distribution.

The normal distribution with $\mu = 0$ and $\sigma = 1$ is called the standard normal distribution. The formula for standardizing a random variable X is:

$$Z = \frac{(X - \mu)}{\sigma}$$

LO.m: Define shortfall risk, calculate the safety-first ratio, and select an optimal portfolio using Roy's safety-first criterion.

Shortfall risk is the risk that portfolio value will fall below some minimum acceptable level over some time horizon.

$$\text{Safety First Ratio or SF-Ratio} = \frac{[E(R_p) - R_L]}{\sigma_p}$$

To select the optimal portfolio according to Roy's criterion, we follow the following steps:

- Calculate each portfolio's SF-Ratio.
- Choose the portfolio with the highest SF-Ratio.

LO.n: Explain the relationship between normal and lognormal distributions and why

the lognormal distribution is used to model asset prices.

If x is a random variable that is normally distributed, then to create a lognormal distribution of x we take e^x and plot the values on a graph.

A lognormal distribution is often used to model asset prices because the asset prices need to be positive, they cannot be negative.

LO.o: Distinguish between discretely and continuously compounded rates of return, and calculate and interpret a continuously compounded rate of return, given a specific holding period return.

Discretely compounded rates of returns have defined compounding periods such as quarterly, monthly. As we decrease the length of the compounding period, the effective annual rate rises. For continuous compounding, the EAR is given by: $EAR = e^r - 1$.

If we are given the holding period return over any time period, we can calculate the equivalent continuously compounded rate of return for that period as: $r = \ln(HPR + 1)$

LO.p: Explain Monte Carlo simulation and describe its applications and limitations.

Monte Carlo simulation is a computer simulation that produces a distribution of possible security prices using randomly generated values for risk factors, based on their assumed distributions.

Limitations include:

- It is fairly complex and will provide answers that are no better than the assumptions.
- Simulation is not an analytical method but a statistical one.

LO.q: Compare Monte Carlo simulation and historical simulation.

The historical simulation uses actual past distribution of risk factors as input. Whereas, Monte Carlo simulation uses randomly generated values of the risk factors based on their assumed distributions.

The limitations of historical simulation are:

- It cannot take into account the effect of significant events that did not occur during the sample period.
- It cannot perform a 'what if' analysis when the 'if' scenario has not happened in the past.

Practice Questions

- Which of the following is *most likely* an example of a discrete random variable?
 - The returns on a portfolio.
 - The time spent by an analyst on research.
 - The number of stocks in a portfolio.
- For a continuous random variable X , the probability of any single value of X is:
 - 0.
 - 1.
 - determined by the cumulative distribution function.

- The probability distribution of a discrete random variable X is given below

X	0	1	2	3	4	5	6
$P(X)$	0.15	0.25	0.12	0.16	0.08	0.13	0.11

What is the cdf of 4 or $F(4)$?

- 0.08.
 - 0.76.
 - 0.68.
- The Cumulative distribution function of a discrete random variable X is given below:

X	Cumulative Distribution Function
0	0.15
1	0.4
2	0.52
3	0.68
4	0.76
5	0.89
6	1

The probability of X greater than or equal to 2 and less than or equal to 5 is *closest* to:

- 0.37.
 - 0.52.
 - 0.89.
- Which of the following is *least likely* a condition of a binomial experiment?
 - The trials are independent.
 - There are only two trials.
 - The probability of success + probability of failure = 1.

6. Castle Hockey Club, an emerging hockey team, had a tough last season with a win to loss record of 1 to 4. In order to contest and increase its chances of winning, the team signed 3 new players and it is estimated that the chances of winning a game in the next season are 70%. Assuming that winning a single game is independent of other games, the probability that the team will win 3 out of next 5 games is *closest* to:
- 0.3087.
 - 0.3698.
 - 0.4125.

7. A stock's predicted price over the next two periods is as shown below.

Time = 0	Time = 1	Time = 2
$S_0 = 100$	$S_u = 104$	$S_{uu} = 108$
	$S_d = 96$	$S_{ud,du} = 101$
		$S_{dd} = 92$

The initial value of the stock is 100. From historical data, it has been observed that the probability of an up move in any given period is 60% and the probability of a down move in any given period is 40%. Using the above data, the probability that the stock's price will be equal to 101 at the end of period 2 is *closest* to:

- 24%.
 - 36%.
 - 48%.
8. For a fixed income arbitrage strategy, the returns for a single trade is uniformly distributed between 3% and 8%. If a new trade is placed today, what is the probability that the returns will be more than 5%?
- 0.4
 - 0.6
 - 0.8
9. For a standard normal distribution $F(0)$ is:
- 0.
 - 0.5
 - 1
10. For a standard normal distribution, $P(0 \leq Z \leq 1.65)$ is:
- 0.45.
 - 0.475.
 - 0.9.
11. Given that X follows a normal distribution with a mean of 5 and standard deviation of

1.25, the standardized value corresponding to $X = 9$ would be *closest* to:

- A. 3.2
- B. 4.
- C. 5.

12. Following data on the three portfolios is given:

Portfolio	Expected Return	Standard Deviation
1	10%	3%
2	15%	5%
3	20%	10%

Assuming the minimum acceptable rate of return is 5%, under Roy's safety-first criterion, which of the following portfolios is the *most* appropriate choice?

- A. Portfolio 1.
- B. Portfolio 2.
- C. Portfolio 3.

13. Analyst 1: Normal distributions are suitable for asset returns.

Analyst 2: Lognormal distributions are suitable for describing distributions of asset prices.

- A. Analyst 1 is correct.
- B. Analyst 2 is correct.
- C. Both analysts are correct.

14. A stock's price at the start of the year was \$10 and its year-end price was \$12, then its continuously compounded annual rate of return is:

- A. 16.45%
- B. 18.23%
- C. 20%

15. Which of the following simulations can be used for 'what if' analysis?

- A. Monte Carlo simulation.
- B. Historical simulation.
- C. Both Monte Carlo and historical simulation.

Solutions

1. C is correct. A discrete random variable is one for which the number of possible outcomes can be counted. A continuous random variable is one for which the number of possible outcomes is infinite. Returns and time are usually continuously random variables. The number of stocks in a portfolio is a discrete random variable.
2. A is correct. For a continuous distribution $p(x) = 0$ for all X ; only ranges of value of X have positive probabilities.
3. B is correct. A cumulative distribution function (cdf) gives the probability that a random variable will be less than or equal to specific values. Therefore: $F(4) = 0.15 + 0.25 + 0.12 + 0.16 + 0.08 = 0.76$.
4. A is correct. $P(X \leq 5) = 0.89$ and $P(X \leq 2) = 0.52$. Therefore, $P(2 \leq X \leq 5) = 0.89 - 0.52 = 0.37$.
5. B is correct. There may be any number of independent trials, each with only two possible outcomes.
6. A is correct. Using Binomial distribution to calculate the probability of 3 wins out of 5 games:
 $P(x) = P(X = x) = [\text{number of ways to choose } x \text{ from } n] \times p^x \times (1 - p)^{n-x}$
 $P(\text{Win}) = 0.7$
 $P(\text{Lose}) = 0.3$
 $P(3 \text{ out of } 5) = {}^5C_3 \times 0.7^3 \times (0.3)^2 = 0.3087$
7. C is correct. There are four possibilities that stock price can take. It can move up-up, up-down, down-up and down-down. In order to reach 101, a stock's price can either go up first and then down or the other way. The probability for the stock going up first and then down is $0.6 \times 0.4 = 0.24$. This probability will be the same for the stock going down first and then up which is, $0.4 \times 0.6 = 0.24$. Therefore, the probability of the stock's price reaching 101 is 48% ($24\% + 24\%$).
8. B is correct. In uniform distribution, the probability can be found of a range only instead of a point. The return is uniformly distributed from 3% to 8%, i.e. the range is 5%. If a new trade is placed then returns greater than 5% leaves a 3% buffer ($8\% - 5\% = 3\%$). Therefore, the probability is $3/5$ or 0.6.
9. B is correct. The normal distribution is symmetric around a mean of 0. $F(0) = 0.5$ since half of the distribution lies on each side of the mean.

10. A is correct. From the table $F(1.65) = 0.95$, and $F(0) = 0.5$. Therefore the answer is $0.95 - 0.5 = 0.45$.

Alternatively, we know that 90% lie within -1.65 and $+1.65$ and that 0 is the midpoint, there we can say that half of 90% i.e 45% lies between 0 and $+1.65$.

11. A is correct. If X follows a normal distribution with parameters μ and σ , X can be standardized using the formula:

$$\begin{aligned} Z &= \frac{X - \mu}{\sigma} \\ &= (9 - 5) \div 1.25 \\ &= 3.2 \end{aligned}$$

12. B is correct. The portfolio with the highest SFRatio is preferred. The SFRatio is calculated by

$$\frac{E(R_p) - R_L}{\sigma_P}$$

The SF ratios for the portfolios are:

Portfolio 1: 1.66

Portfolio 2: 2

Portfolio 3: 1.5

13. C is correct. A normal distribution is suitable for describing asset returns. However, the normal distribution is not suitable for asset prices because asset prices cannot be negative. The lognormal distribution is bounded by zero (skewed to the right) and is suitable for describing distributions of asset prices.

14. B is correct.

$\ln(\text{Ending Price} / \text{Starting Price})$

$$\ln(12/10) = 0.1823 = 18.23\%$$

15. A is correct. Compared with Monte Carlo simulation, the historical simulation cannot be used for a “what if” analysis.

R11 Sampling and Estimation

1. Introduction

A sample is a subset of a population. We can study a sample to infer conclusions about the population itself. For example, if all the stocks trading in the US are considered a population, then indices such as the S&P 500 are samples. We can look at the performance of the S&P 500 and draw conclusions about how all stocks in the US are performing. This process is known as sampling and estimation.

2. Sampling

There are various methods for obtaining information on a population through samples. The information we obtain usually concerns a **parameter**, a quantity used to describe a population. To estimate a parameter, we use sample statistics. A **statistic** is a quantity used to describe a sample.

There are two reasons why sampling is used:

- Time saving: In many cases it will be very time consuming to examine every member of the population.
- Monetary saving: In some cases, examining every member of the population becomes economically inefficient.

2.1. Simple Random Sampling

Simple random sampling is the process of selecting a sample from a larger population in such a way that each member of the population has the same probability of being included in the sample.

Sampling distribution

If we draw samples of the same size several times and calculate the sample statistic, the sample statistic will be different each time. The distribution of values of the sample statistic is called a sampling distribution.

For example, say you select 100 stocks from a universe of 10,000 stocks and calculate the average annual returns of these 100 stocks. Let's say you get an average return of 15%. You repeat this process with a second sample of 100 stocks. This time, you get an average return of 14%. You keep repeating this process and each time you get a different average return. The distribution of these sample average returns is called a sampling distribution.

Sampling error

Sampling error is the difference between a sample statistic and the corresponding population parameter. The sampling error of the mean is given by:

$$\text{Sampling error of the mean} = \bar{x} - \mu$$

For example, let's say you want to estimate the average returns of 10,000 stocks. You draw a

sample of 100 stocks and calculate the average return of these 100 stocks as 15%. However, the actual average of the 10,000 stocks was 12%. Then the sampling error = $15\% - 12\% = 3\%$.

2.2. Stratified Random Sampling

In stratified random sampling, the population is divided into sub groups based on one or more distinguishing characteristics. Samples are then drawn from each sub group, with sample size proportional to the size of the sub group relative to the population. Finally, samples from each sub group are pooled together to form a stratified random sample.

The advantage of stratified random sampling is that the sample will have the same distribution of key characteristics as the overall population. This can help reduce the sampling error.

For example, you divide the universe of 10,000 stocks as per their market capitalization such that you have 5,000 large cap stocks, 3,000 mid cap stocks, and 2,000 small cap stocks. In stratified random sampling, to select a total sample of 100 stocks, you will randomly select 50 large cap stocks, 30 mid cap stocks, and 20 small cap stocks and pool all these samples together to form a stratified random sample.

Example

Paul wants to categorize publicly listed stocks for his research project. He first divides the stocks into 15 industries. Then from each industry, he categorizes companies into three groups: small, medium, large. Finally, he divides these into value versus growth stocks. How many cells or strata does the sampling plan entail?

- A. 20
- B. 45
- C. 90

Solution:

C is correct. This is an application of the multiplication rule of counting. The total number of cells is the product of 15, 3, and 2. Thus the answer is 90.

2.3. Time-Series and Cross-Sectional Data

Time series data

Time series data consists of observations for a single subject taken at specific and equally spaced intervals of time. For example, the monthly returns on Microsoft stock from January 1995 to January 2005.

Cross sectional data

Cross-sectional data consists of observations for multiple subjects taken at a specific point in time. For example, the sample of reported earnings per share of all NASDAQ companies for

2005.

For both time series and cross-sectional data, the random sample must be representative of the population we wish to study.

3. Distribution of the Sample Mean

The sample mean is a random variable with a probability distribution known as the statistic's sampling distribution. To understand this concept, consider the following population: last year's returns on every stock traded in the United States. We are interested in the mean return of all stocks but do not have time to calculate the population mean. Hence, we draw a sample of 50 stocks and compute the sample mean. We then draw another sample of 50 stocks and compute the sample mean. This exercise can be repeated several times giving us a distribution of sample means. This distribution is called the statistic's sampling distribution. The central limit theorem, explained below, helps us understand the sampling distribution of the mean.

3.1. The Central Limit Theorem

According to the central limit theorem, if we draw a sample from a population with a mean μ and a variance σ^2 , then the sampling distribution of the sample mean:

- will be normally distributed (irrespective of the type of distribution of the original population).
- will have a mean of μ .
- will have a variance of σ^2/n .

For example, suppose the average return of the universe of 10,000 stocks is 12% and its standard deviation is 10%. Through central limit theorem we can conclude that if we keep drawing samples of 100 stocks and plot their average returns, we will get a sampling distribution that will be normally distributed with mean = 12% and variance of $10^2/100 = 1\%$.

Standard error of the sample mean

The standard deviation of the distribution of the sample means is known as the standard error of the sample mean.

When we know the population standard deviation, the standard error of the sample mean can be calculated as:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

When we do not know the population standard deviation (σ) we can use the sample standard deviation (s) to estimate the standard error of the sample mean:

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

Example

The mean of a population is 12 and the standard deviation is 3. Given that the population comprises of 64 observations, what is the standard error of the sample mean?

- A. 0.375
- B. 0.378
- C. 0.667

Solution:

A is correct. Standard Error = $\sigma/\sqrt{n} = 3/\sqrt{64} = 0.375$

4. Point and Interval Estimates of the Population Mean

Statistical inference consists of two branches: 1) hypothesis testing and 2) estimation. Hypothesis testing addresses the question 'Is the value of this parameter equal to some specific value?' This is discussed in detail in the next reading. In this section, we will discuss estimation. Estimation seeks to answer the question 'What is this parameter's value?' In estimation, we make the best use of the information in a sample to form one of several types of estimates of the parameter's value.

A **point estimate** is a single number that estimates the unknown population parameter.

Interval estimates (or confidence intervals) are a range of values that bracket the unknown population parameter with some specified level of probability.

4.1. Point Estimators

The formulas that we use to compute the sample mean and all other sample statistics are examples of estimation formulas or **estimators**. The particular value that we calculate from sample observations using an estimator is called an **estimate**. For example, the calculated value of the sample mean in a given sample is called a **point estimate** of the population mean.

The three desirable properties of an estimator are:

- **Unbiasedness:** Its expected value is equal to the parameter being estimated.
- **Efficiency:** It has the lowest variance as compared to other unbiased estimators of the same parameter.
- **Consistency:** As sample size increases, the sampling error decreases and the estimates get closer to the actual value.

4.2. Confidence Intervals for the Population Mean

A confidence interval is a range of values, within which the actual value of the parameter will lie with a given probability. Confidence interval is calculated as:

$$\text{Confidence interval} = \text{point estimate} \pm (\text{reliability factor} \times \text{standard error})$$

where:

point estimate = a point estimate of the parameter

reliability factor = a number based on the assumed distribution of the point estimate and the degree of confidence $(1 - \alpha)$ for the confidence interval

standard error = standard error of the sample statistic providing the point estimate

Calculating confidence intervals

To calculate a confidence interval for a population mean, refer to the table below and select t statistic or z statistic as per the scenario.

Sampling from		Small sample size($n < 30$)	Large sample size($n \geq 30$)
Normal distribution	Variance known	z	z
	Variance unknown	t	t (or z)
Non -normal distribution	Variance known	NA	z
	Variance unknown	NA	t (or z)

Use the following formulae to calculate the confidence interval:

$$\text{Confidence interval} = \bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$\text{Confidence interval} = \bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

Sampling from a normal distribution with known variance

The most basic confidence interval for the population mean arises when we are sampling from a normal distribution with known variance.

The confidence interval will be calculated as:

$$\text{Confidence Interval} = \bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

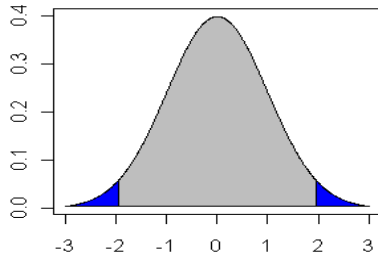
where:

\bar{X} = sample mean

$z_{\alpha/2}$ = reliability factor

σ/\sqrt{n} = standard error

The reliability factor ($z_{\alpha/2}$) depends purely on the degree of confidence. If the degree of confidence $(1 - \alpha)$ is 95% or 0.95, the level of significance (α) is 5% or 0.05. $\alpha/2 = 0.025$. $\alpha/2$ is the probability in each tail of the standard normal distribution. This is shown in blue in the figure below:



The reliability factors that are most frequently used when we construct confidence intervals based on the standard normal distribution are:

- 90% confidence intervals: $\alpha = 0.1$, $\alpha/2 = 0.05$. Reliability factor = $z_{0.05} = 1.65$
- 95% confidence intervals: $\alpha = 0.05$, $\alpha/2 = 0.025$. Reliability factor = $z_{0.025} = 1.96$
- 99% confidence intervals: $\alpha = 0.01$, $\alpha/2 = 0.005$. Reliability factor = $z_{0.005} = 2.58$

Memorize these confidence intervals and the corresponding reliability factors.

Example

You take a random sample of stocks on the National Stock Exchange (NSE). The sample size is 100 and the average Sharpe ratio is 0.50. Assume that the Sharpe ratios of all stocks on the NSE follow a normal distribution with a standard deviation of 0.30. What is the 90% confidence interval for the mean Sharpe ratio of all stocks on the NSE?

Solution:

$$\text{Confidence Interval} = \bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 0.50 \pm 1.65 \frac{0.30}{\sqrt{100}} = 0.50 \pm 0.0495$$

Therefore, the 90% confidence interval for the mean Sharpe ratio of all stocks on the NSE is: 0.4505 to 0.5495.

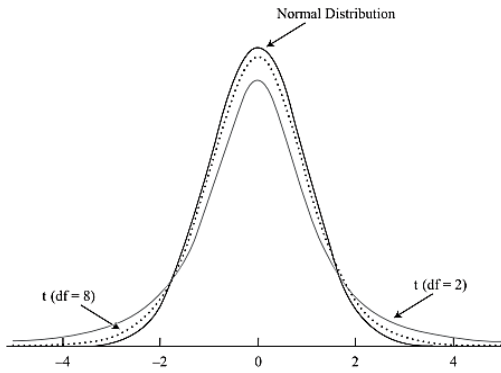
As we increase the degree of confidence, the confidence interval becomes wider. If we use a 95% confidence interval, the reliability factor is 1.96. And the confidence interval ranges from $0.50 - 1.96 \times 0.03$ to $0.50 + 1.96 \times 0.03$ which is: 0.4412 to 0.5588. This range is wider than what we had with a 90% confidence interval.

Sampling from a normal distribution with unknown variance

If the distribution of the population is normal with unknown variance, we can use the t-distribution to construct a confidence interval. Let us first understand what a t-distribution is and contrast it with a normal distribution.

Student's t-distribution has the following properties:

- It is symmetrical, bell-shaped and similar to a normal distribution.
- It has a lower peak and fatter tails as compared to a normal distribution.
- It is defined by a single parameter, degrees of freedom ($df = n - 1$).
- As the degrees of freedom increase, the shape of the t-distribution starts approaching the shape of the normal distribution.



The confidence interval can be calculated using the following formula:

$$\text{Confidence Interval} = \bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

This relationship is similar to what we've discussed before except here we use the t-distribution. Since the population standard deviation is not known, we have to use the sample standard deviation which is denoted by the symbol 's'. We will now see how to read the t-distribution table using an example.

Example

Given a sample size of 20, what is the reliability factor for a 90% confidence level?

Solution:

In order to answer this question, we need to refer to the t-table. A snapshot of the table is given below. This table shows the level of significance for one-tailed probabilities.

df	P = 0.10	P = 0.05	P = 0.025	P = 0.01	P = 0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921

17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845

The degrees of freedom in this case are $n - 1 = 20 - 1 = 19$. The level of significance is calculated as: $100(1 - \alpha) \% = 90\%$ and $\alpha = 0.10$. In order to calculate the reliability factor $t_{\alpha/2} = t_{0.10/2} = t_{0.05}$, we look at the row with $df = 19$. We then look at the column with $p = 0.05$. The value that satisfies these two criteria is 1.729. This is the reliability factor.

Example

An analyst wants to estimate the return on the Hang Seng Index for the current year using the following data and assumptions:

- Sample size = 64 stocks from the index
- Mean return for the stocks in the sample for the previous year = 0.12
- Variance = 0.081

It is given that the reliability factor for a 95% confidence interval with unknown population variance and sample size greater than 64 is 1.96. Assuming that the index return this year will be the same as it was last year, what is the estimate of the 95% confidence interval for the index's return this year?

Solution:

$$\text{Confidence Interval} = \bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 0.12 \pm 1.96 \left[\frac{\sqrt{0.081}}{\sqrt{64}} \right] = 0.12 \pm 0.07 = 0.05 \text{ to } 0.19$$

4.3. Selection of Sample Size

The choice of sample size affects the width of a confidence interval. A larger sample size decreases the width of a confidence interval and improves the precision with which we can estimate the population parameter. This is obvious when we consider the confidence interval relationship, shown below:

$$\text{Confidence Interval} = \bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

All else equal, when the sample size (n) is increased, the standard error (s/\sqrt{n}) decreases and we have a more precise (narrower) confidence interval. Based on this discussion it might appear that we should use as large a sample size as possible. However, we must consider the following issues:

- Increasing the sample size may result in sampling from more than one population.
- Increasing the sample size may involve additional expenses that outweigh the value of additional precision.

5. More on Sampling

There are many issues in sampling. Here we discuss four main issues.

5.1 Data-Mining Bias

Data-mining is the practice of analyzing the same data again and again, till a pattern that works is identified.

There are two signs that can warn analysts about the potential existence of data mining:

- Too much digging/too little confidence: Many different variables were tested, until significant ones were found. Unfortunately, many researchers do not disclose the number of variables examined in developing a model.
- No story/no future: Without a reasonable economic rationale or story for why a variable should work, the variable is unlikely to have predictive power.

The best way to avoid this bias is to test the pattern on out of sample data to check if it holds.

5.2 Sample Selection Bias

When data availability leads to certain assets being excluded from the analysis, the resulting problem is known as sample selection bias. For example, many mutual fund databases provide historical information about only those funds that currently exist. A type of sample selection bias is the survivorship bias, in which companies are excluded from analysis because they have gone out of business. A sample can also be biased because of the removal (or delisting) of a company's stock from an exchange.

5.3 Look-Ahead Bias

A test design is subject to look-ahead bias if it uses information that was not available to market participants at the time the market participants act in the model. For example, an analyst wants to use the company's book value per share to construct the P/B variable for that particular company. Although the market price of a stock is available for all market participants at the same point in time, fiscal year-end book equity per share might not become publicly available until sometime in the following quarter.

5.4 Time-Period Bias

A test design is subject to time-period bias if it is based on a time period that may make the results time-period specific. A short time series is likely to give period specific results that may not reflect a longer period. If a time series is too long, fundamental structural changes may have occurred in that time period.

Summary

LO.a: Define simple random sampling and a sampling distribution.

Simple random sampling is the process of selecting a sample from a larger population in such a way that each element of the population has the same probability of being included in the sample.

If we draw samples of the same size several times and calculate the sample statistic. The sample statistic will be different each time. The distribution of values of the sample statistic is called a sampling distribution.

LO.b: Explain sampling error.

Sampling error is the difference between a sample statistic and the corresponding population parameter.

Sampling error of the mean = $\bar{x} - \mu$

LO.c: Distinguish between simple random and stratified random sampling.

In stratified random sampling, the population is divided into sub groups based on one or more distinguishing characteristics. Samples are then drawn from each sub group, with sample size proportional to the size of the sub group relative to the population. Finally, samples from each sub group are pooled together to form a stratified random sample.

LO.d: Distinguish between time-series and cross-sectional data.

Time series data consists of observations for a single subject taken at specific and equally spaced intervals of time.

Cross-sectional data consists of observations for multiple subjects taken at a specific point in time.

LO.e: Explain the central limit theorem and its importance.

According to the central limit theorem, if we draw a sample from a population with a mean μ and a variance σ^2 , then the sampling distribution of the sample mean:

- will be normally distributed (irrespective of the type of distribution of the original population).
- will have a mean of μ .
- will have a variance of σ^2/n .

LO.f: Calculate and interpret the standard error of the sample mean.

The standard deviation of the distribution of the sample means is known as the standard error of the sample mean.

When we know the population standard deviation, the standard error of the sample mean can be calculated as: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

When we do not know the population standard deviation (σ) we can use the sample standard deviation (s) to estimate the standard error of the sample mean $s_{\bar{x}} = \frac{s}{\sqrt{n}}$

LO.g: Identify and describe desirable properties of an estimator.

The three desirable properties of an estimator are:

- **Unbiasedness:** Its expected value is equal to the parameter being estimated.
- **Efficiency:** It has the lowest variance as compared to other unbiased estimators of the same parameter.
- **Consistency:** As sample size increases, the sampling error decreases and the estimates get closer to the actual value.

LO.h: Distinguish between a point estimate and a confidence interval estimate of a population parameter.

A point estimate is a single value estimate that serves as an approximation for the actual value of the parameter.

A confidence interval is a range of values, within which the actual value of the parameter will lie with a given probability.

Confidence interval = Point estimate \pm (reliability factor \times standard error of point estimate)

LO.i: Describe properties of Student's t-distribution and calculate and interpret its degrees of freedom.

Student's t-distribution has the following properties:

- It is symmetrical, bell-shaped and similar to a normal distribution.
- It has a lower peak and fatter tails as compared to a normal distribution.
- It is defined by a single parameter, degrees of freedom (df) = $n-1$
- As the degrees of freedom increase, the shape of the t-distribution starts approaching the shape of the normal distribution.

LO.j: Calculate and interpret a confidence interval for a population mean, given a normal distribution with 1) a known population variance, 2) an unknown population variance, or 3) an unknown population variance and a large sample size.

Refer to the table below and select t statistic or z statistic as per the scenario.

Sampling from		Small sample size($n < 30$)	Large sample size($n \geq 30$)
Normal distribution	Variance known	z	z
	Variance unknown	t	t (or z)
Non –normal distribution	Variance known	NA	z
	Variance unknown	NA	t (or z)

Use the following formulae to calculate the confidence interval:

$$\text{Confidence interval} = \bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$\text{Confidence interval} = \bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

LO.k: Describe the issues regarding selection of the appropriate sample size, data-mining bias, sample selection bias, survivorship bias, look-ahead bias, and time-period bias.

Increasing the sample size reduces the standard error and gives us narrower confidence intervals. However, while increasing sample size we must consider two things:

- Cost involved: Compare the cost of getting more data to the potential benefits of increasing precision.
- Risk of sampling from a different population: In the process of increasing sample size if we get data from a different population, then the accuracy will not improve.

Biases observed in sampling methods are:

- Data-mining bias: Analyzing the same data again and again, till a pattern that works is identified. The best way to avoid this bias is to test the pattern on out of sample data to check if it holds.
- Sample selection bias: Excluding certain assets from the analysis due to unavailability of data. A type of sample selection bias is the *survivorship bias*, in which companies are excluded from analysis because they have gone out of business and data for them was not easily available.
- Look-ahead bias: Using information that became available at later periods in time in the analysis.
- Time-period bias: If the selected time period is too short, the results may only hold for that time period. If the time period is too long, then the results might not consider major structural changes in the economy.

Practice Questions

1. A simple random sample is a sample drawn in such a way that each member of the population has:
 - A. some chance of being selected in the sample.
 - B. an equal chance of being selected in the sample.
 - C. is included in the sample.
2. Suppose we take a random sample of 40 companies in an industry with 100 companies. We calculate the sample mean of the debt-to-equity ratio for the prior year. We find that this ratio is 2.5 percent. Subsequently, we learn that the population debt-to-equity ratio (taking account of all 100 companies) is 2. What is the explanation for the discrepancy between the sample mean of 2.5 percent and the population mean of 2?
 - A. Sampling error.
 - B. Bias.
 - C. A lack of consistency.
3. An analyst compiles EPS data for a sample of companies randomly drawn from the S&P 500. He gathers the data from the companies' fiscal year 2015 annual reports. This data is *best* characterized as:
 - A. time-series data.
 - B. longitudinal data.
 - C. cross-sectional data.
4. The population distribution of hedge fund returns in a country is found to be negatively skewed in a particular year. Given that the sample size is large, the sampling distribution of the sample mean returns would be characterized by:

	Distribution	Mean	Variance
A	Negative skewed	Less than population mean	Equal to population variance
B	Normal distribution	Equal to population mean	Less than population variance
C	Normal distribution	Less than population mean	Equal to population variance

5. A sample of 50 observations has a mean of 10 and a standard deviation of 2. The standard error of the sample mean is *closest* to:
 - A. 0.04
 - B. 0.28.
 - C. 1.41.

6. An analyst while generating an estimate of a population parameter is considering drawing a larger sample size. In doing so, a larger sample size will *most likely* improve the estimator's:
 - A. efficiency.
 - B. consistency.
 - C. unbiasedness.
 7. John Clark is unsure of the underlying population distribution and wants to increase the reliability of the parameter estimates. Which of the following is *least likely* to achieve his objective?
 - A. Increase in the sample size.
 - B. Use of point estimates rather than confidence intervals.
 - C. Use of the t-distribution rather than the normal distribution to establish confidence intervals.
 8. Analyst 1: As degrees of freedom increase, a t-distribution will become more peaked.
Analyst 2: As degrees of freedom increase, the tails of the t-distribution will become less fat.
 - A. Analyst 1 is correct.
 - B. Analyst 2 is correct.
 - C. Both analysts are correct.
 9. Robert Scott wants to calculate the confidence intervals for the population mean of an approximately normal distribution. Assuming the sample size is small, the Student's t-distribution is the preferred method when the variance is:
 - A. unknown.
 - B. too Large.
 - C. negative.
 10. A random sample of 100 coffee shop customers spent an average of 25 mins at the store. Assuming the distribution is normal and the population standard deviation is 4 min, the 95% confidence interval for the population mean is *closest* to:
 - A. 21 to 29.
 - B. 24.25 to 25.78.
 - C. 23.78 to 26.04.
 11. A sample of 25 recent orders at a restaurant showed that the mean time to serve a dish was 19 mins with a sample standard deviation of 3 mins. Assuming the population is normally distributed, the 99% confidence interval for the population mean is:
 - A. 17.32 to 20.67
 - B. 16.21 to 21.96.
-

C. 15.24 to 22.38.

12. David Jones is conducting a research on S&P 500 stocks. In his research, he uses historical data that was not publicly available at the time period being studied. His sample will *most likely* have a:

- A. look-ahead bias.
- B. time-period bias.
- C. sample selection bias.

Solutions

1. B is correct. In a simple random sample, each member of the population has an equal probability of being selected.
2. A is correct. The discrepancy arises from sampling error. Sampling error is the difference between a sample statistic and its corresponding population parameter. A sampling error exists whenever one fails to observe every element of the population, because a sample statistic can vary from sample to sample. We do not expect the sample mean to exactly equal the population mean in any one sample we may take.
3. C is correct. Data on some characteristics of companies at a single point in time are cross-sectional data.
4. B is correct. According to the Central Limit Theorem, the sampling distribution of sample means is approximately a normal distribution with its mean is equal to the population mean. Its variance is equal to population variance divided by the sample size and hence would be less than population variance. The Central Limit Theorem is independent of the probability distribution of the given population when the sample size is large and thus the skewness of population does not matter.
5. B is correct. The standard error of the sample mean, when we know the sample standard deviation, is sample standard deviation divided by square root of sample size:
 $2/\sqrt{50} = 0.28$
6. B is correct. Unbiasedness and efficiency are properties of an estimator's sampling distribution that hold for any size sample. A consistent estimator is one for which the probability of estimates close to the value of the population parameter increases as sample size increases.
7. B is correct. Point estimates are not as reliable as confidence interval estimates. A conservative approach to confidence intervals relies on the *t*-distribution rather than the normal distribution, and use of the *t*-distribution will increase the reliability of the confidence interval. Increasing the sample size increases the reliability of the confidence intervals.
8. C is correct. As degrees of freedom increase, the *t*-distribution will more closely resemble a normal distribution, becoming more peaked and having less fat tails.
9. A is correct. The Student's *t*-distribution is the preferred option when the sample size is small and the variance is unknown
Below table provides the appropriate test given a scenario:

Sampling From	Statistic for Small Sample Size	Statistic for Large Sample Size
Normal distribution with a known variance	z	z
Normal distribution with unknown variance	t	t or z
Non-normal distribution with known variance	Not Available	z
Non-normal distribution with unknown variance	Not Available	t or z

10. B is correct. Since the population variance is known and $n \geq 30$, the confidence interval is determined as $\bar{x} \pm Z_{\alpha/2}(\sigma/\sqrt{n})$ $Z_{0.025} = 1.96$

Confidence interval = $25 \pm 1.96 (4/\sqrt{100}) = 24.216$ to 25.784

11. A is correct. Since the population variance is unknown and $n < 30$, the confidence interval is determined as

$\bar{x} \pm t_{\alpha/2}(s/\sqrt{n})$. The critical value of $t_{0.005}$ and $df = n-1 = 24$ is 2.797.

So the confidence interval is $19 \pm 2.797 (3/\sqrt{5}) = 17.32$ to 20.67

12. A is correct. This is an example of look ahead bias.

R12 Hypothesis Testing

1. Introduction

Hypothesis testing is the process of making judgments about a larger group (a population) on the basis of observing a smaller group (a sample). The results of such a test then help us evaluate whether our hypothesis is true or false.

For example, let's say you are a researcher and you believe that the average return on all Asian stocks was greater than 2%. To test this belief you can draw samples from a population of all Asian stocks and employ hypothesis testing procedures. The results of this test can tell you if your belief is statistically valid.

2. Hypothesis Testing

A hypothesis is defined as a statement about one or more populations. In order to test a hypothesis, we follow these steps:

1. Stating the hypothesis.
2. Identifying the appropriate test statistic and its probability distribution.
3. Specifying the significance level.
4. Stating the decision rule.
5. Collecting the data and calculating the test statistic.
6. Making the statistical decision.
7. Making the economic or investment decision.

Instructor's Note

Though the curriculum suggests a seven step process for hypothesis testing, you can arrive at a decision quickly using these four steps:

1. State the hypothesis.
2. Compute the test statistic.
3. Determine the critical value based on significance level.
4. Compare the test statistic with the critical value and determine whether or not to reject the null hypothesis.

Let's understand these steps with the help of a few examples.

Right Tailed Test

Suppose you are a researcher and believe that the average return on all Asian stocks was greater than 2%. In this case, you are making a statement about the population mean (μ) of all Asian stocks.

Step 1: Stating the hypothesis

The first step is stating the two hypotheses: the null hypothesis (H_0) and the alternative hypothesis (H_a).

Null hypothesis (H_0): It is the hypothesis that the researcher wants to reject.

Alternative hypothesis (H_a): It is the hypothesis that the researcher wants to prove. If the null hypothesis is rejected then the alternative hypothesis is considered valid.

For our example, the null and alternative hypotheses are:

$$H_0: \mu \leq 2\%$$

$$H_a: \mu > 2\%$$

(The value 2% is known as μ_0 , the hypothesized value of the population mean.)

Instructor's Note:

An easy way to differentiate between the two hypotheses is to remember that the null hypothesis always contains some form of the equal sign.

Step 2: Compute the test statistic.

A test statistic is calculated from sample data and is compared to a critical value to decide whether or not we can reject the null hypothesis. The formula for computing test statistic is:

$$\text{test statistic} = \frac{\text{sample statistic} - \text{value of the parameter under } H_0}{\text{standard error of the sample statistic}}$$

Continuing with our example, let's further suppose that the sample mean of 36 observations of Asian stocks is 4 and the standard deviation of the population is 4. Therefore,

$$\text{test statistic} = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{4 - 2}{\frac{4}{\sqrt{36}}} = 3$$

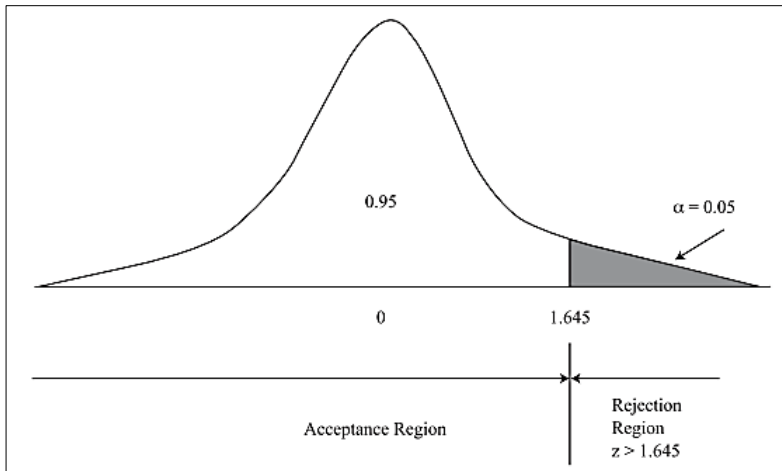
Step 3: Determine the critical value based on significance level

Continuing with our example of Asian stocks, suppose we want to test our hypothesis at the 5% significance level. This is a one-tailed test because we are trying to assess whether the population mean is greater than 2% or not. Hence, we are only interested in the right tail of the distribution. If we were trying to assess whether the population mean is less than 2% we would be interested in the left tail.

Instructor's Note

The sign in the alternative hypothesis points to the direction of the tail that we should use in our test. Since in our example the alternative hypothesis has a '>' sign it points to the right, therefore we are interested in the right tail.

The critical value is also known as the **rejection point** for the test statistic. Graphically, this point separates the acceptance and rejection regions for a set of values of the test statistic. This is shown below:



The region to the left of the test statistic is the 'acceptance region'. This represents the set of values for which we do not reject (accept) the null hypothesis. The region to the right of the test statistic is known as the 'rejection region'.

Using the Z -table and 5% level of significance, the critical value = $Z_{0.05} = 1.65$

Step 4: Compare the test statistic with the critical value and determine whether or not to reject the null hypothesis

If the test statistic > critical value, i.e. if the test statistic lies in the rejection region we will reject H_0 .

On the other hand, if the test statistic < critical value, i.e. if the test statistic lies in the acceptance region, then we cannot reject H_0 .

In our example, because the test statistic $z = 3$ is greater than the critical value of 1.645, we reject the null hypothesis in favor of the alternative hypothesis that the average return on all Asian stocks is greater than 2%.

Left Tailed Test

We use a left tailed test to determine whether the estimated value of a population parameter is less than a hypothesized value.

Example

An analyst believes that the average return on all Asian stocks was *less* than 2%. The sample size is 36 observations with a sample mean of -3. The standard deviation of the population is 4. Will he reject the null hypothesis at the 5% level of significance?

Solution:

In this case, our null and alternative hypotheses are:

$$H_0: \mu \geq 2$$

$$H_a: \mu < 2$$

The standard error of the sample is: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{36}} = 0.67$

The test statistic is:

$$\text{test statistic} = \frac{\text{sample statistic} - \text{value of the parameter under } H_0}{\text{standard error of the sample statistic}} = \frac{-3 - 2}{0.67} = -7.5$$

The critical values corresponding to a 5% level of significance is -1.65.

When we consider the left tail of the distribution, our decision rule is as follows: Reject the null hypothesis if the test statistic is less than the critical value and vice versa. Since our calculated test statistic of -7.5 is less than the critical value of -1.65, we reject the null hypothesis.

Two-tailed test

We use a two-tailed hypothesis test to determine whether the estimated value of a population parameter is different from the hypothesized value. In a two-tailed test, we reject the null in favor of the alternative if the evidence indicates that the population parameter is either smaller or larger than the value of the parameter under H_0 .

For example, we believe that the average return on all Asian stocks was not 0%. We take a sample of 36 observations with a sample mean of 1 and a population standard deviation of 4. In this case our null and alternative hypotheses are:

$$H_0: \mu = 0$$

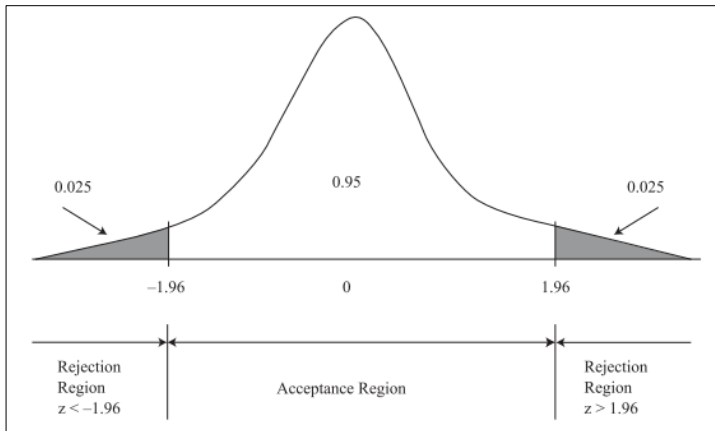
$$H_a: \mu \neq 0$$

The standard error of the sample is unchanged at 0.67: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{36}} = 0.67$

The test statistic is:

$$\text{test statistic} = \frac{\text{sample statistic} - \text{value of the parameter under } H_0}{\text{standard error of the sample statistic}} = \frac{1 - 0}{0.67} = 1.5$$

In a two-tailed test, two critical values exist, one positive and one negative. For a two-sided test at the 5% level of significance, we calculate the z-values that correspond to $\frac{0.05}{2} = 0.025$ level of significance. These are +1.96 and -1.96. Therefore, we reject the null hypothesis if we find that the test statistic is less than -1.96 or greater than +1.96. We fail to reject the null hypothesis if $-1.96 \leq \text{test statistic} \leq +1.96$. Graphically, this can be shown as:



Relationship between confidence interval and hypothesis tests

The above figure also illustrates the relationship between confidence intervals and hypothesis tests. A confidence interval specifies the range of values that may contain the hypothesized value of the population parameter. The 5% level of significance in the hypothesis tests corresponds to a 95% confidence interval. When the hypothesized value of the population parameter is outside the corresponding confidence interval, the null hypothesis is rejected. When the hypothesized value of the population parameter is inside the corresponding confidence interval, the null hypothesis is not rejected.

Summary: One-tailed test versus two-tailed tests

One-tailed tests are used to test if the population parameter is greater than or less than a hypothesized value. They can be either right-tailed tests or left-tailed tests.

Two-tailed tests are used to test if the population parameter is not equal to the hypothesized value. They have two rejection points: one positive and one negative, because here we are interested in both tails.

p-value

The p-value is the smallest level of significance at which the null hypothesis can be rejected. It can be used in the hypothesis testing framework as an alternative to using rejection points.

- If the p-value is lower than our specified level of significance, we reject the null hypothesis.
- If the p-value is greater than our specified level of significance, we do not reject the null hypothesis.

For example, if the p-value of a test is 4%, then the hypothesis can be rejected at the 5% level of significance, but not at the 1% level of significance.

Relationship between test-statistic and p-value

A high test-statistic implies a low p-value.

A low test-statistic implies a high p-value.

Type I and Type II errors

In reaching a statistical decision, we can make two possible errors:

- Type I error: We may reject a true null hypothesis.
- Type II error: We may fail to reject a false null hypothesis.

The following table shows the possible outcomes of a test.

Decision	True condition	
	H ₀ true	H ₀ false
Do not reject H ₀	Correct decision	Type II error
Reject H ₀ (accept H _a)	Type I error	Correct decision

Power of a test

The power of a test is the probability of correctly rejecting the null (rejecting the null when it is false). It is expressed as:

$$\text{Power of a test} = 1 - P(\text{Type II error})$$

Statistical result v/s economically meaningful result

A statistical decision simply consists of rejecting or not rejecting the null hypothesis. Whereas, an economic decision takes into consideration all economic issues relevant to the decision, such as transaction costs, risk tolerance and the impact on the existing portfolio. Sometimes a test may indicate a statistically significant result which may not be economically significant.

3. Hypothesis Tests Concerning the Mean

3.1. Tests Concerning a Single Mean

One of the decisions we need to make in hypothesis testing is deciding which test statistic and which corresponding probability distribution to use. We use the following table to make this decision:

Sampling from		Small sample size (n<30)	Large sample size (n≥30)
Normal distribution	Variance known	z	z
	Variance unknown	t	t (or z)
Non-normal distribution	Variance known	NA	z
	Variance unknown	NA	t (or z)

In the hypothesis tests we saw so far, the population variance was known and our sample size was large (n≥30), hence we used the z statistic and z-distribution to compute the critical value.

However, if we do not know the population variance and we have a small sample size, then we have to use the t statistic and t distribution to compute the critical values.

Example

Fund Alpha has been in existence for 20 months and has achieved a mean monthly return of 2% with a sample standard deviation of 5%. The expected monthly return for a fund of this nature is 1.60%. Assuming monthly returns are normally distributed, are the actual results consistent with an underlying population mean monthly return of 1.60%?

Solution:

The null and alternative hypotheses for this example will be:

$H_0: \mu = 1.60$ versus $H_a: \mu \neq 1.60$

$$\text{test statistic} = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{2 - 1.60}{\frac{5}{\sqrt{20}}} = 0.36$$

Using this formula, we see that the value of the test statistic is 0.36.

The critical values at a 0.05 level of significance can be calculated from the t-distribution table. Since this is a two-tailed test, we should look at a $0.05/2 = 0.025$ level of significance with $df = n - 1 = 20 - 1 = 19$. This gives us two values of -2.1 and +2.1.

Since our test statistic of 0.35 lies between -2.1 and +2.1, i.e., the acceptance region, we do not reject the null hypothesis.

3.2. Tests Concerning Differences between Means**Instructor's Note:**

Focus on the basics of this topic, the probability of being tested on the details is low.

In this section, we will learn how to calculate the difference between the means of two independent and normally distributed populations. We can use two kinds of t-tests. In one case the population variances, although unknown, can be assumed to be equal. In the second case the population variances are assumed to be unknown and unequal.

Unknown But Equal Variance

When we assume that the two populations are normally distributed and that the unknown population variances are equal, the t-test based on independent random samples is given by:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{(\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2})^{1/2}}$$

The term s_p^2 is known as the pooled estimator of the common variance. It is calculated by the following formula:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

The number of degrees of freedom is $n_1 + n_2 - 2$.

Unknown and Unequal Variance

When we can assume that the two populations are normally distributed and that the unknown population variances are unequal, an approximate t-test based on independent random samples is given by:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^{1/2}}$$

In this formula, we use the tables of the t-distribution using the 'modified' degrees of freedom. The 'modified' degrees of freedom are calculated using the following formula:

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1} + \frac{(s_2^2/n_2)^2}{n_2}}$$

Example

You believe the mean return on NYSE stocks was different from the mean on NSE stocks last month. To test your hypothesis you collect the following data:

	Sample Size (n)	Sample Mean (\bar{X})	Sample Standard Deviation (s)
NSE	20	2%	4
NYSE	40	3%	5

Determine whether to reject the null hypothesis at the 0.10 level of significance.

Solution:

The first step is to formulate the null and alternative hypotheses. Since we want to test whether the two means were equal or different, we define the hypotheses as:

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_a: \mu_1 - \mu_2 \neq 0$$

Since the population standard deviation is unknown and we cannot assume that it is equal, we use the following formula to calculate the test statistic:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^{1/2}} = \frac{(2 - 3) - (0)}{\left(\frac{4^2}{20} + \frac{5^2}{40}\right)^{1/2}} = -0.84$$

Next, we calculate the modified degrees of freedom:

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1} + \frac{(s_2^2/n_2)^2}{n_2}} = \frac{\left(\frac{4^2}{20} + \frac{5^2}{40}\right)^2}{\frac{(4^2/20)^2}{20} + \frac{(5^2/40)^2}{40}} = 48$$

For a 0.10 level of significance, we find the t-value for $0.10/2 = 0.05$ using $df = 48$. The t-

value is therefore $t_{\alpha/2} = -1.677$ and $+1.677$. Since our test statistic of -0.84 lies in the acceptance region, we fail to reject the null hypothesis.

3.3. Tests concerning Mean Differences

Instructor's Note:

Focus on the basics of this topic, the probability of being tested on the details is low.

In the previous section, in order to perform hypothesis tests on differences between means of two populations, we assumed that the samples were independent. What if the samples are not independent? For example, suppose you want to conduct tests on the mean monthly return on Toyota stock and mean monthly return on Honda stock. These two samples are believed to be dependent, as they are impacted by the same economic factors.

In such situations, we conduct a t-test that is based on data arranged in **paired observations**. Paired observations are observations that are dependent because they have something in common.

We will now discuss the process for conducting such a t-test. Suppose that we gather data regarding the mean monthly returns on stocks of Toyota and Honda for the last 20 months, as shown in the table below:

Month	Mean return of Toyota stock	Mean monthly return of Honda stock	Difference in mean monthly returns (d_i)
1	0.5%	0.4%	0.1%
2	0.7%	1.0%	-0.3%
3	0.3%	0.7%	-0.4%
...
20	0.9%	0.6%	0.3%
Average	0.750%	0.600%	0.075%

Here is a simplified process for conducting the hypothesis test:

Step 1: Define the null and alternate hypotheses

We believe that the mean difference is not 0. Hence the null and alternate hypotheses are:

$$H_0: \mu_d = \mu_{d0} \text{ versus } H_a: \mu_d \neq \mu_{d0}$$

μ_d stands for the population mean difference and μ_{d0} stands for the hypothesized value for the population mean difference.

Step 2: Calculate the test-statistic

Determine the sample mean difference using:

$$\bar{d} = \frac{1}{n} \sum_{i=0}^n d_i$$

For the data given, the sample mean difference is 0.075%.

Calculate the sample standard deviation. The process for calculating the sample standard deviation has been discussed in an earlier reading. The simplest method is to plug the numbers (0.1, -0.3, -0.4...0.3) into a financial calculator. The entire data set has not been provided. We'll take it as a given that the sample standard deviation is 0.150%.

Use this formula to calculate the standard error of the mean difference:

$$s_{\bar{d}} = \frac{s_d}{\sqrt{n}}$$

For our data this is $0.150 / \sqrt{20} = 0.03354$.

We now have the required data to calculate the test statistic using a t-test. This is calculated using the following formula using $n - 1$ degrees of freedom:

$$t = \frac{\bar{d} - \mu_{d0}}{s_{\bar{d}}}$$

For our data, the test statistic is $\frac{0.075 - 0}{0.03354} = 2.24$.

Step 3: Determine the critical value based on the level of significance

We will use a 5% level of significance. Since this is a two-tailed test we have a probability of 2.5% (0.025) in each tail. This critical value is determined from a t-table using a one-tailed probability of 0.025 and $df = 20 - 1 = 19$. This value is 2.093.

Step 4: Compare the test statistic with the critical value and make a decision

In our case, the test statistic (2.23) is greater than the critical value (2.093). Hence we will reject the null hypothesis.

Conclusion: The data seems to indicate that the mean difference is not 0.

The hypothesis test presented above is based on the belief that the population mean difference is not equal to 0. If μ_{d0} is the hypothesized value for the population mean difference, then we can formulate the following hypotheses:

11. If we believe the population mean difference is greater than 0:
 $H_0: \mu_d \leq \mu_{d0}$ versus $H_a: \mu_d > \mu_{d0}$:
12. If we believe the population mean difference is less than 0:
 $H_0: \mu_d \geq \mu_{d0}$ versus $H_a: \mu_d < \mu_{d0}$
13. If we believe the population mean difference is not 0:
 $H_0: \mu_d = \mu_{d0}$ versus $H_a: \mu_d \neq \mu_{d0}$

4. Hypothesis Tests Concerning Variance

Instructor's Note:

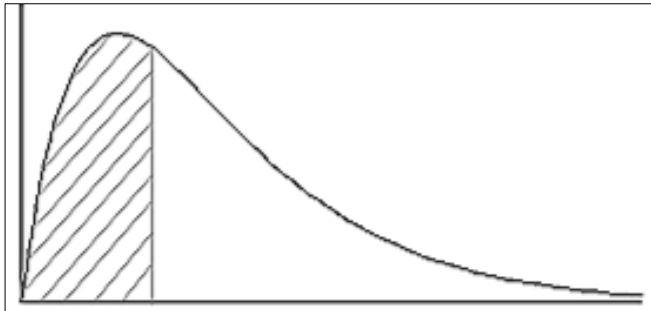
Focus on the basics of this topic, the probability of being tested on the details is low.

4.1. Tests Concerning a Single Variance

In tests concerning the variance of a single normally distributed population, we use the chi-square test statistic, denoted by χ^2 .

Properties of the chi-square distribution

The chi-square distribution is asymmetrical and like the t-distribution, is a family of distributions. This means that a different distribution exists for each possible value of degrees of freedom, $n - 1$. Since the variance is a squared term, the minimum value can only be 0. Hence, the chi-square distribution is bounded below by 0. The graph below shows the shape of a chi-square distribution:



There are three hypotheses that can be formulated (σ^2 represents the true population variance and σ_0^2 represents the hypothesized variance):

1. $H_0: \sigma^2 = \sigma_0^2$ versus $H_a: \sigma^2 \neq \sigma_0^2$. This is used when we believe the population variance is not equal to 0, or it is different from the hypothesized variance. It is a two-tailed test.
2. $H_0: \sigma^2 \geq \sigma_0^2$ versus $H_a: \sigma^2 < \sigma_0^2$. This is used when we believe the population variance is less than the hypothesized variance. It is a one-tailed test.
3. $H_0: \sigma^2 \leq \sigma_0^2$ versus $H_a: \sigma^2 > \sigma_0^2$. This is used when we believe the population variance is greater than the hypothesized variance. It is a one-tailed test.

After drawing a random sample from a normally distributed population, we calculate the test statistic using the following formula using $n - 1$ degrees of freedom:

$$\chi^2 = \frac{(n - 1)(s^2)}{\sigma_0^2}$$

where:

n = sample size

s = sample variance

We then determine the critical values using the level of significance and degrees of freedom. The chi-square distribution table is used to calculate the critical value.

Example

Consider Fund Alpha which we discussed in an earlier example. This fund has been in existence for 20 months. During this period the standard deviation of monthly returns was 5%. You want to test a claim by the fund manager that the standard deviation of monthly returns is less than 6%.

Solution:

The null and alternate hypotheses are: $H_0: \sigma^2 \geq 36$ versus $H_a: \sigma^2 < 36$

Note that the standard deviation is 6%. Since we are dealing with population variance, we will square this number to arrive at a variance of 36%.

We then calculate the value of the chi-square test statistic:

$$\chi^2 = (n - 1) s^2 / \sigma_0^2 = 19 \times 25/36 = 13.19$$

Next, we determine the rejection point based on $df = 19$ and significance = 0.05. Using the chi-square table, we find that this number is 10.117.

Since the test statistic (13.19) is higher than the rejection point (10.117) we cannot reject H_0 . In other words, the sample standard deviation is not small enough to validate the fund manager's claim that population standard deviation is less than 6%.

4.2. Tests Concerning the Equality (Inequality) of Two Variances

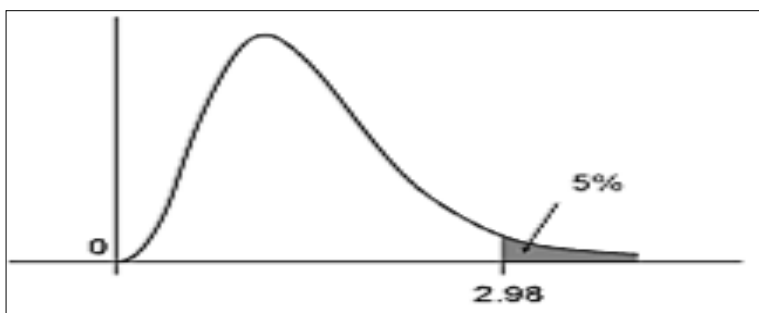
In order to test the equality or inequality of two variances, we use an F-test which is the ratio of sample variances.

The assumptions for a F-test to be valid are:

- The samples must be independent.
- The populations from which the samples are taken are normally distributed.

Properties of the F-distribution

The F-distribution, like the chi-square distribution, is a family of asymmetrical distributions bounded from below by 0. Each F-distribution is defined by two values of degrees of freedom, called the numerator and denominator degrees of freedom. As shown in the figure below, the F-distribution is skewed to the right and is truncated at zero on the left hand side.



As shown in the graph, the rejection region is always in the right side tail of the distribution.

When working with F-tests, there are three hypotheses that can be formulated:

1. $H_0: \sigma_{12} = \sigma_{22}$ versus $H_a: \sigma_{12} \neq \sigma_{22}$. This is used when we believe the two population variances are not equal.
2. $H_0: \sigma_{12} \leq \sigma_{22}$ versus $H_a: \sigma_{12} > \sigma_{22}$. This is used when we believe the variance of the first population is greater than the variance of the second population.
3. $H_0: \sigma_{12} \geq \sigma_{22}$ versus $H_a: \sigma_{12} < \sigma_{22}$. This is used when we believe the variance of the first population is less than the variance of the second population.

The term σ_1^2 represents the population variance of the first population and σ_2^2 represents the population variance of the second population.

The formula for the test statistic of the F-test is:

$$F = \frac{s_1^2}{s_2^2}$$

where:

s_1^2 = the sample variance of the first population with n observations

s_2^2 = the sample variance of the second population with n observations

A convention is to put the larger sample variance in the numerator and the smaller sample variance in the denominator.

$df_1 = n_1 - 1$ numerator degrees of freedom

$df_2 = n_2 - 1$ denominator degrees of freedom

The test statistic is then compared with the critical values found using the two degrees of freedom and the F-tables.

Finally, a decision is made whether to reject or not to reject the null hypothesis.

Example

You are investigating whether the population variance of the Indian equity market changed after the deregulation of 1991. You collect 120 months of data before and after deregulation. Variance of returns before deregulation was 13. Variance of returns after deregulation was 18. Check your hypothesis at a confidence level of 99%.

Solution:

Null and alternate hypothesis: $H_0: \sigma_1^2 = \sigma_2^2$ versus $H_A: \sigma_1^2 \neq \sigma_2^2$

F-statistic: $\frac{18}{13} = 1.4$

df = 119 for the numerator and denominator

$\alpha = 0.01$ which means 0.005 in each tail. From the F-table: critical value = 1.6

Since the F-stat is less than the critical value, do not reject the null hypothesis.

Summary of types of test statistics

Hypothesis test of	Use
One population mean	t-statistic or z-statistic
Two population mean	t-statistic
One population variance	Chi-square statistic
Two-population variance	F-statistic

5. Other Issues: Nonparametric Inference

The hypothesis-testing procedures we have discussed so far have two characteristics in common:

- They are concerned with parameters, such as the mean and variance.
- Their validity depends on a set of assumptions.

Any procedure which has either of the two characteristics is known as a **parametric test**.

Nonparametric tests are not concerned with a parameter and/or make few assumptions about the population from which the sample are drawn. We use nonparametric procedures in three situations:

- Data does not meet distributional assumptions.
- Data are given in ranks. (Example: relative size of the company and use of derivatives.)
- The hypothesis does not concern a parameter. (Example: Is a sample random or not?)

The Spearman rank correlation coefficient test is a popular nonparametric test. The coefficient is calculated based on the ranks of two variables within their respective samples.

Summary

LO.a: Define a hypothesis, describe the steps of hypothesis testing, and describe and interpret the choice of the null and alternative hypotheses.

A hypothesis is a statement about the value of a population parameter developed for the purpose of testing a theory.

In order to test a hypothesis, we follow these steps:

1. State the hypothesis.
2. Identify the appropriate test statistic.
3. Specify the level of significance.
4. State a decision rule to accept or reject the hypothesis.
5. Collect sample data and calculate the test statistic.
6. Decide if the hypothesis can be accepted/ rejected.
7. Make an economic or investment decision.

The null hypothesis (H_0) is the hypothesis that the researcher wants to reject. It should always include some form of the 'equal to' condition.

The alternative hypothesis (H_a) is the hypothesis that the researcher wants to prove. If the null hypothesis is rejected then the alternative hypothesis is considered valid.

LO.b: Distinguish between one-tailed and two-tailed tests of hypotheses.

In one-tailed tests, we are assessing if the value of a population parameter is greater than or less than a hypothesized value.

In two-tailed tests, we are assessing if the value of a population parameter is different from a hypothesized value.

LO.c: Explain a test statistic, Type I and Type II errors, a significance level, and how significance levels are used in hypothesis testing.

A test statistic is a quantity, calculated on the basis of a sample, and is used to decide whether to reject or not to reject the null hypothesis. The formula for computing the test statistic is:

$$\text{test statistic} = \frac{\text{sample statistic} - \text{value of the parameter under } H_0}{\text{standard error of the sample statistic}}$$

In reaching a statistical decision, we can make two possible errors: We may reject a true null hypothesis (a Type I error), or we may fail to reject a false null hypothesis (a Type II error).

The level of significance of a test is the probability of a Type I error.

As α gets smaller the critical value gets larger and it becomes more difficult to reject the null hypothesis.

LO.d: Explain a decision rule, the power of a test, and the relation between confidence

intervals and hypothesis tests.

A decision rule consists of comparing the computed test statistic to the critical values (rejection points) based on the level of significance to decide whether to reject or not to reject the null hypothesis.

The power of a test is the probability of correctly rejecting the null (rejecting the null when it is false). It is expressed as:

$$\text{Power of a test} = 1 - P(\text{Type II error})$$

A confidence interval gives us the range of values within which a population parameter is expected to lie. Confidence intervals and hypothesis tests are linked through critical values. The null hypothesis will be rejected only if the test statistic lies outside the confidence interval.

LO.e: Distinguish between a statistical result and an economically meaningful result.

The statistical decision consists of rejecting or not rejecting the null hypothesis. The economic decision takes into consideration all economic issues relevant to the decision.

LO.f: Explain and interpret the p-value as it relates to hypothesis testing.

The p-value is the smallest level of significance at which the null hypothesis can be rejected. It can be used in the hypothesis testing framework as an alternative to using rejection points.

- If the p-value is lower than our specified level of significance, we reject the null hypothesis.
- If the p-value is greater than our specified level of significance, we do not reject the null hypothesis.

LO.g: Identify the appropriate test statistic and interpret the results for a hypothesis test concerning the population mean of both large and small samples when the population is normally or approximately distributed and the variance is 1) known or 2) unknown.

We use the following table to decide which test statistic and which corresponding probability distribution to use for hypothesis testing.

Sampling from		Small sample size ($n < 30$)	Large sample size ($n \geq 30$)
Normal distribution	Variance known	z	z
	Variance unknown	t	t (or z)
Non-normal distribution	Variance known	NA	z
	Variance unknown	NA	t (or z)

LO.h: Identify the appropriate test statistic and interpret the results for a hypothesis test concerning the equality of the population means of two at least approximately normally distributed populations, based on independent random samples with 1)

equal or 2) unequal assumed variances.

When we can assume that the two populations are normally distributed and that the unknown population variances are equal, the t-test based on independent random samples is given by:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{(\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2})^{1/2}}$$

The number of degrees of freedom is $n_1 + n_2 - 2$. The term s_p^2 is known as the pooled estimator of the common variance. A **pooled estimate** is an estimate drawn from the combination of two different samples. It is calculated as:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

When we can assume that the two populations are normally distributed and that the unknown population variances are unequal, an approximate t-test based on independent random samples is given by:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2})^{1/2}}$$

In this formula, we use the tables of the t-distribution using the 'modified' degrees of freedom. The 'modified' degrees of freedom are calculated as:

$$df = \frac{(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2})^2}{\frac{(s_1^2/n_1)^2}{n_1} + \frac{(s_2^2/n_2)^2}{n_2}}$$

LO.i: Identify the appropriate test statistic and interpret the results for a hypothesis test concerning the mean difference of two normally distributed populations.

In cases where we have a test concerning the mean difference of two normally distributed populations that are dependent, we conduct a t-test that is based on data arranged in paired observations.

The hypothesis is formed on the difference between means of two populations e.g. $H_0: \mu_d = \mu_{d0}$ versus $H_a: \mu_d \neq \mu_{d0}$

In order to arrive at the test statistic, we first determine the sample mean difference using:

$$\bar{d} = \frac{1}{n} \sum_{i=0}^n d_i$$

And the standard error of the mean difference is computed as follows:

$$s_{\bar{d}} = \frac{s_d}{\sqrt{n}}$$

Once we have these two values, we can calculate the test statistic using a t-test. This is calculated using the following formula using $n - 1$ degrees of freedom:

$$t = \frac{\bar{d} - \mu_{d0}}{s_{\bar{d}}}$$

The value of calculated test statistic is compared with the t-distribution values in the usual manner to arrive at a decision on our hypothesis.

LO.j: Identify the appropriate test statistic and interpret the results for a hypothesis test concerning 1) the variance of a normally distributed population, and 2) the equality of the variances of two normally distributed populations based on two independent random samples.

In tests concerning the variance of a single normally distributed population, we use the chi-square test statistic, denoted by χ^2 . After drawing a random sample from a normally distributed population, we calculate the test statistic using the following formula using $n - 1$ degrees of freedom:

$$\chi^2 = \frac{(n - 1)(s^2)}{\sigma_0^2}$$

We then determine the critical values using the level of significance and degrees of freedom. The chi-square distribution table is used to calculate the critical value.

In order to test the equality or inequality of two variances, we use an F-test. The critical value is computed as:

$$F = \frac{s_1^2}{s_2^2}$$

The test statistic is then compared with the critical values found using the two degrees of freedom and the F-tables. Finally, a decision is made whether to reject or not to reject the null hypothesis.

LO.k: Distinguish between parametric and nonparametric tests and describe situations in which the use of nonparametric tests may be appropriate.

A parametric test is a hypothesis test concerning a parameter or a hypothesis test based on specific distributional assumptions. In contrast, a nonparametric test is either not concerned with a parameter or makes minimal assumptions about the population from which the sample is drawn.

A nonparametric test is primarily used in three situations: when data do not meet distributional assumptions, when data is given in ranks, or when the hypothesis we are addressing does not concern a parameter.

Practice Questions

1. David Jones is a researcher and wants to test if the mean returns of his bond investments is more than 5% per year. In this case, the null and alternative hypothesis would be *best* defined as:
 - A. $H_0: \mu = 5$ versus $H_a: \mu \neq 5$.
 - B. $H_0: \mu \leq 5$ versus $H_a: \mu > 5$.
 - C. $H_0: \mu \geq 5$ versus $H_a: \mu < 5$.
2. Which of the following statements requires a two-tailed test?
 - A. $H_0: \mu \leq 0$ versus $H_a: \mu > 0$.
 - B. $H_0: \mu \geq 0$ versus $H_a: \mu < 0$.
 - C. $H_0: \mu = 0$ versus $H_a: \mu \neq 0$.
3. All else equal, specifying a smaller significance level in a hypothesis test will *most likely* increase the probability of:
 - A. type I error.
 - B. type II error.
 - C. both Type I and Type II errors.
4. All else equal, increasing the sample size for a hypothesis test will *most likely* decrease the probability of:
 - A. type I error.
 - B. type II error.
 - C. both Type I and Type II errors.
5. If the significance level of a test is 0.05 and the probability of a Type –II error is 0.2. What is the power of the test?
 - A. 0.05.
 - B. 0.80.
 - C. 0.95.
6. A researcher formulates a null hypothesis that the mean of a distribution is equal to 10. He obtains a p-value of 0.02. Using a 5% level of significance, the *best* conclusion is to:
 - A. reject the null hypothesis.
 - B. accept the null hypothesis.
 - C. decrease the level of significance.
7. Which of the following statistic is *most likely* used for the mean of a non-normal distribution with unknown variance and a small sample size?
 - A. z-test statistic.

- B. t-test statistic.
 - C. There is no test statistic for such a scenario.
8. You believe that the average returns of all stocks in the S&P 500 is greater than 10%. You draw a sample of 49 stocks. The average return of these 49 stocks is 12%. The standard deviation of returns of all stocks in the S&P 500 is 4. Using a 5% level of significance, which of the following conclusions is *most appropriate*?
- A. We can conclude that the average returns of all stocks in the S&P 500 is greater than 10%.
 - B. We can conclude that the average returns of all stocks in the S&P 500 is less than 10%.
 - C. We can conclude that the average returns of all stocks in the S&P 500 is equal to 10%.
9. The appropriate test statistic to test the hypothesis that the variance of a normally distributed population is equal to 8 is the:
- A. t-test.
 - B. F-test.
 - C. χ^2 test.

Solutions

1. B is correct. The null hypothesis is what the researcher wants to reject. The alternative hypothesis is what the researcher wants to prove, and it is accepted when the null hypothesis is rejected.
2. C is correct. A two-tailed test for the population mean is structured as: $H_0: \mu = 0$ versus $H_a: \mu \neq 0$.
3. B is correct. Specifying a smaller significance level decreases the probability of a Type I error (rejecting a true null hypothesis), but increases the probability of a Type II error (not rejecting a false null hypothesis). As the level of significance decreases, the null hypothesis is less frequently rejected.
4. C is correct. The only way to avoid the trade-off between the two types of errors is to increase the sample size; increasing sample size (all else equal) reduces the probability of both types of errors.
5. B is correct. The power of a test = $1 - P(\text{Type II error}) = 1 - 0.2 = 0.8$
6. A is correct. The p-value for a hypothesis is the smallest significance level for which the hypothesis would be rejected. As the p-value is less than the stated level of significance, we reject the null hypothesis.
7. C is correct. The statistic for a small sample size of a non-normal distribution with unknown variance is not available. z-test statistic is used for a large sample size of a non-normal distribution with known variance while t-test statistic is used for large sample size of a non-normal distribution with unknown variance.

Sampling From		Small Sample Size	Large Sample Size
Normal Distribution	Variance known	z	z
	Variance unknown	t	t (or z)
Non -normal Distribution	Variance known	NA	z
	Variance unknown	NA	t (or z)

8. A is correct.
Step 1: State the hypothesis
 $H_0: \mu \leq 10\%$
 $H_a: \mu > 10\%$

Step 2: Calculate the test statistic

The population variance is known hence we will use z-statistic.

$$z - \text{statistic} = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{12 - 10}{\frac{4}{\sqrt{49}}} = 3.5$$

Step 3: Calculate the critical value

This is a one-tailed test and we will be looking at the right tail. Using the Z -table and 5% level of significance

Critical value = $Z_{0.05} = 1.65$

Step 4: Decision

Since the test statistic (3.5) > critical value (1.65), we reject H_0 . Hence at 5% level of significance, your belief that the average returns of all stocks in the S&P 500 is greater than 10% is correct.

9. C is correct. In tests concerning the variance of a single normally distributed population, we use the chi-square test statistic, denoted by χ^2 .

Types of Test Statistics

Hypothesis test of	Use
One population mean	t-statistic or z-statistic
Two population mean	t-statistic
One population variance	Chi-square statistic
Two-population variance	F-statistic

R13 Technical Analysis

1. Introduction

Technical analysis has long been used by traders and analysts, but has only recently received acceptance from regulators and the academic community. Although certain aspects of technical analysis, such as the calculation of indicators, have specific rules, the interpretation of findings is often subjective.

2. Technical Analysis: Definition and Scope

Technical analysis is a form of security analysis that uses price and volume data in decision making. The data is often graphically displayed.

Technical analysis is based on the following logic:

- Supply and demand determine prices.
- Changes in supply and demand can cause changes in prices.
- Prices can be projected with charts and other technical tools.

Technical analysis of any financial instrument does not require detailed knowledge of that instrument. Investors and traders often prefer technical analysis over fundamental analysis in the short term because fundamental analysis is more time consuming.

2.1. Principles and Assumptions

Fundamental theorists believe that markets are rational and efficient. On the other hand, technicians believe that human behavior is often erratic and driven by emotion. Therefore, they believe that market trends and patterns reflect this irrational human behavior. They rely on recognition of such trends and patterns in the past to project future patterns of security prices.

Another principle of technical analysis is that the market reflects the collective knowledge and sentiment of many participants such as investors, hedgers, insiders and other stakeholders. Market participants, who buy and sell securities, have an impact on price. The greater the volume of a participant's trades, the more impact the participant will have on price.

By studying market technical data (price and volume trends), technicians seek to understand investor sentiment.

Some financial instruments have an associated income stream that adds to the security's intrinsic value such as coupon payments for bonds and dividends for equity shares. A fundamental analyst can use these cash flows to arrive at a present value of the security. However, for other assets such as commodities, which do not have underlying income streams or financial statements, technical analysis is commonly used.

2.2. Technical and Fundamental Analysis

Technical and fundamental analyses approach the market in different ways, but are both useful. Key differences between the two are listed below:

- Technicians focus solely on analyzing markets and the trading of financial instruments. Fundamental analysis, on the other hand, focuses on financial and economic analysis as well as societal and political trends analysis.
- Technical analysis is based on price and volume data, while fundamental analysts incorporate external market data, with company's financial statements to predict security price movements. It is important to note that the company's financial statements are often subjective, while price and volume data are objective.
- Fundamental analysis is considered a more theoretical approach, while technical analysis is considered a more practical approach.

Drawbacks of technical analysis are:

- Technicians are limited to studying market movements and do not use other predictive analytical methods.
- Market movements and trends may take some time to become evident. Therefore, technicians may be late in identifying changes in trends and patterns.
- In illiquid markets and markets that are subject to large outside manipulation (such as actions of central banks), the application of technical analysis is limited.

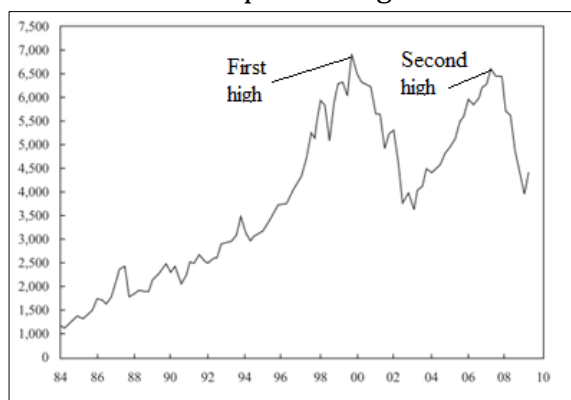
3. Technical Analysis Tools

3.1. Charts

The various types of charts used in technical analysis are:

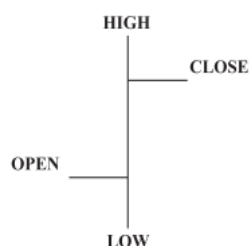
Line chart

Line charts are a simple graphic display of price trends over time. Usually, the chart is a plot of data points, such as share price, with a line connecting these points. The vertical axis (y axis) represents price level and the horizontal axis (x axis) is time. An example of a line chart is shown below. The line chart shows that the price of the stock during the first upward movement of the price is higher than the second high.



Bar chart

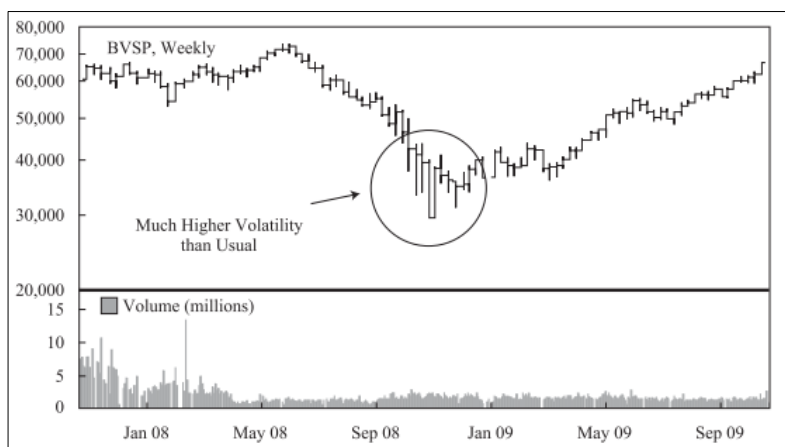
A line chart has only one data point per time interval. A bar chart, in contrast, has four bits of data in each entry - the opening and closing prices, and the high and low prices during the period.



Bar chart notation

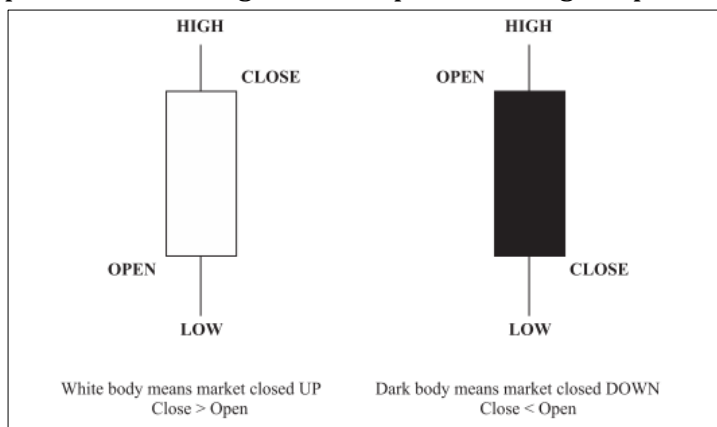
The figure on the left shows how the data point for each interval is constructed. A vertical line connects the high and low price for the day. A cross-hatch to the right indicates the closing price, and a cross-hatch to the left indicates the opening price.

A short bar indicates little price movement while a long bar indicates a wide divergence between the high and the low for the day. An example of a bar chart is shown in the figure below:

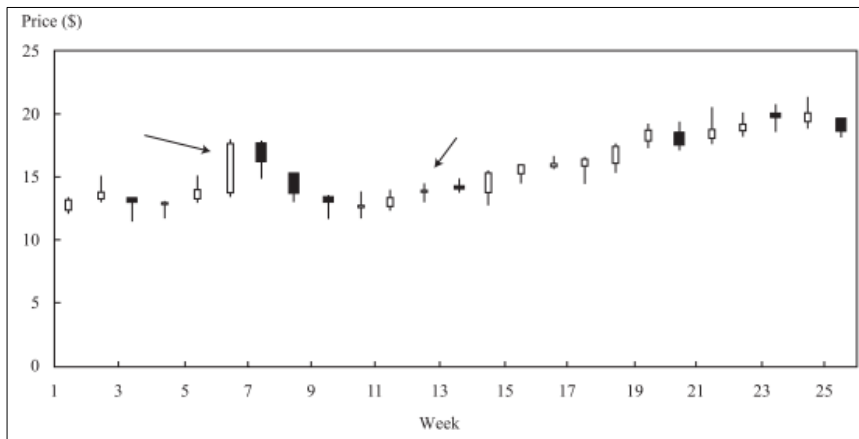


Candlestick chart

A candlestick chart also provides four prices per data entry point: the opening and closing prices, and the high and low prices during the period.



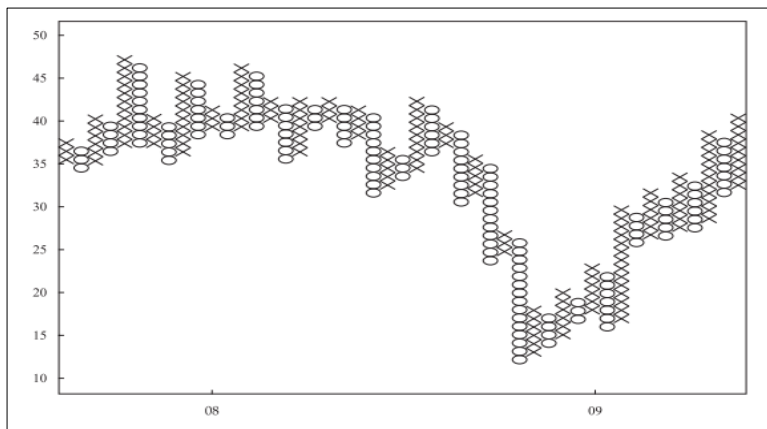
The body of the candle is either white or shaded. A white body means that the market closed up. A shaded body means that the market closed down. An example of a candlestick chart is shown below:



An advantage of the candlestick chart over the bar chart is that price moves are much more visible, which allows for faster analysis.

Point and figure chart

A point and figure chart is drawn on a grid and consists of columns of X's alternating with columns of O's. X represents an increase in price while an O represents a decrease in price. Neither time nor volume is represented on this chart. Instead, the focus is on change in prices. An example of a point and figure chart is given below:



The following steps need to be followed in order to construct a point and figure chart:

- The analyst must first determine the box size. Box size refers to the change in price represented by the height of each box.
- Next, the analyst must determine the reversal size. If there are no significant changes in the stock price, the analyst stays in the same column. Only if there is a substantial reversal, the analyst moves to another column.
- A technician fills in a box with X every time the security's price closes up by the amount of the box size. If the price increases by twice the box size, the technician fills in two

boxes, one on top of the other. If the price does not increase by at least the box size, no indication is made on the chart.

- For price decreases, a technician uses the reversal size to determine when to make an indication on the chart. If our reversal size is 3, the box size is \$1, and there is a price decline of \$3, then the technician will shift to the next column and begin a column of O's.

Point and figure charts are particularly useful for making trading decisions because they clearly illustrate price levels that may signal the end of a decline or advance.

Volume charts

They are often displayed below a line, bar or candlestick chart. The number of units of the security traded is plotted on the Y-axis and time on the X-axis.

Relative strength analysis

Relative strength analysis is used to compare the performance of a particular asset, such as a stock, with that of some benchmark index or the performance of another stock. Typically, the analyst prepares a line chart of the ratio of the two prices, with the asset under analysis as the numerator and the benchmark or other security as the denominator. A rising line shows that the asset is performing better than the benchmark; a declining line shows that the asset is underperforming. A flat line shows neutral performance.

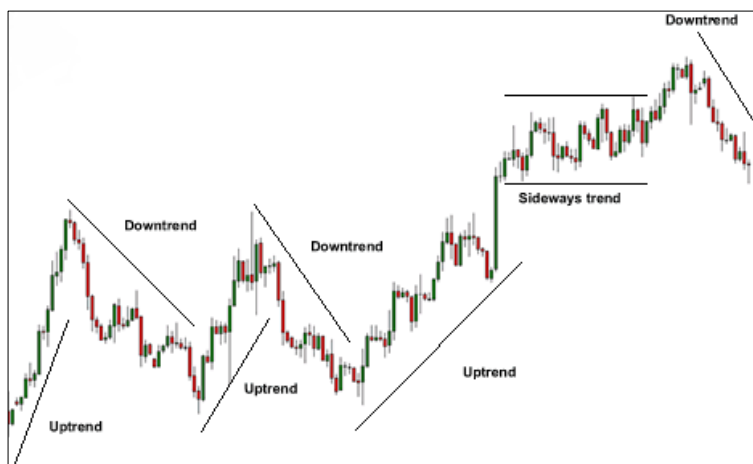
3.2. Trend

Uptrend

A security is said to be in an uptrend if prices are reaching higher highs and higher lows. An upward trendline can be drawn by connecting the increasing low points with a straight line.

Downtrend

A security is said to be in a downtrend if prices are reaching lower highs and lower lows. A downward trendline can be drawn by connecting the decreasing high points with a straight line.



Support

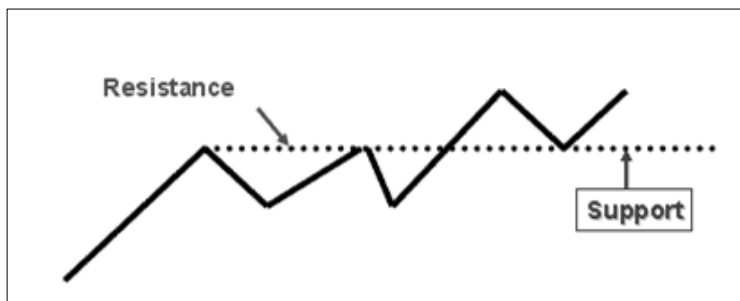
It is the price level at which there is sufficient buying pressure to stop a further decline in prices.

Resistance

It is the price level at which there is sufficient selling pressure to stop a further increase in prices.

Change in polarity

Once a support level is breached, it often becomes a new resistance level. Similarly, once a resistance level is breached, it often becomes a new support level.



3.3. Chart Patterns

Chart patterns are formations that create a recognizable shape; common patterns appear repeatedly and lead to similar subsequent price movements. Chart patterns can be divided into two categories: reversal patterns and continuation patterns.

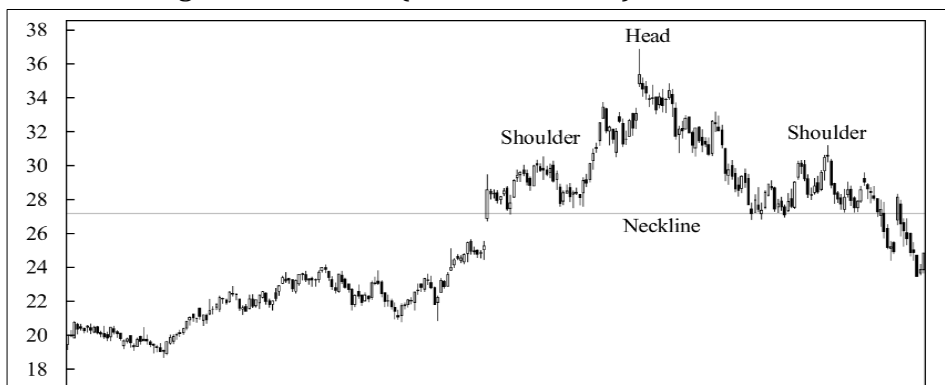
Reversal patterns

They signal the end of a trend. The four kinds of reversal patterns are:

Head and shoulders pattern:

It consists of the left shoulder, the head, and the right shoulder. This pattern indicates the end of an uptrend. You can profit by going short on the security, the price target is:

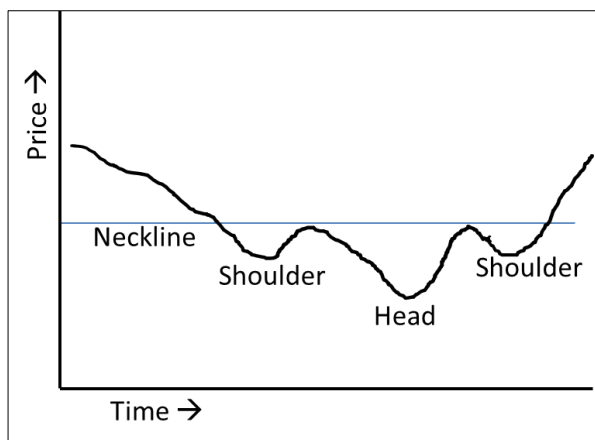
$$\text{Price target} = \text{neckline} - (\text{head} - \text{neckline})$$



Inverse head and shoulders pattern:

It is a mirror image of the head and shoulders pattern. This pattern indicates the end of a downtrend. You can profit by going long on the security, the price target is:

$$\text{Price target} = \text{neckline} + (\text{head} - \text{neckline})$$

**Example**

In an inverted head and shoulders pattern, if the neckline is at €125, the shoulders at €80, and the head at €95, the price target is *closest* to which of the following?

- A. €155.
- B. €110.
- C. €95.

Solution:

Inverted Head and shoulder patter target price = Neckline + (Neckline – Head)

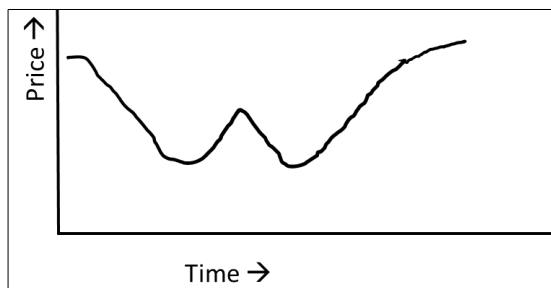
$$\text{Target Price} = 125 + (125 - 95)$$

$$\text{Target Price} = 155$$

Double tops and bottoms:

A double top is formed when prices hit the same resistance level twice and fall down. It indicates the end of an uptrend.

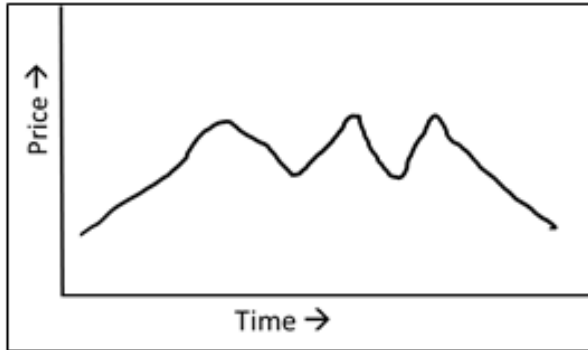
A double bottom is formed when prices bounce back from the same support level twice. It indicates the end of a down-trend.



Triple tops and bottoms:

Triple tops are formed when prices hit the same resistance level thrice.

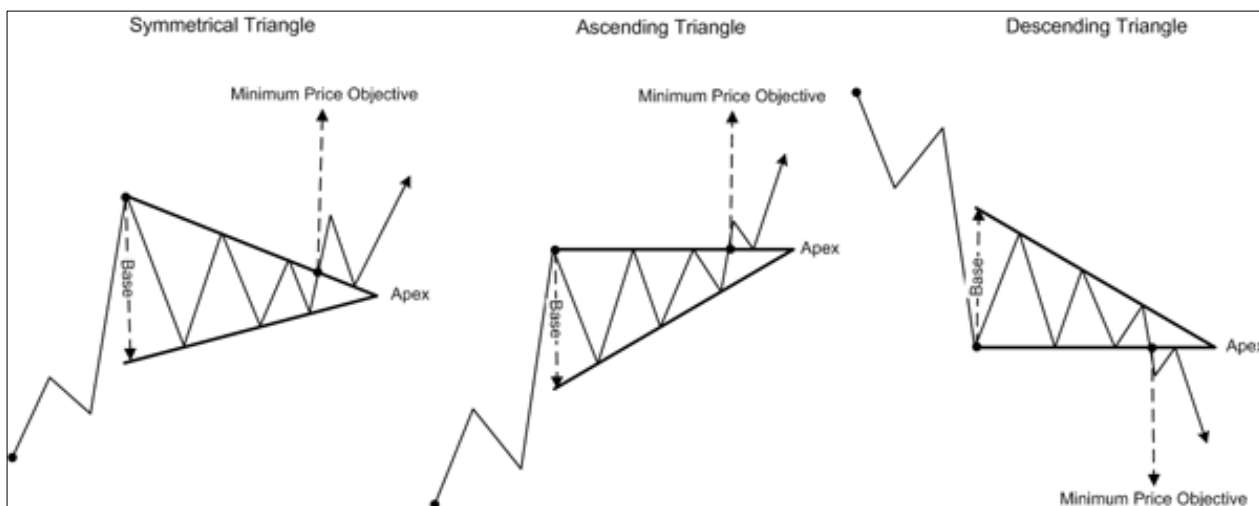
Triple bottoms are formed when prices bounce back from the same support level thrice.

**Continuation patterns**

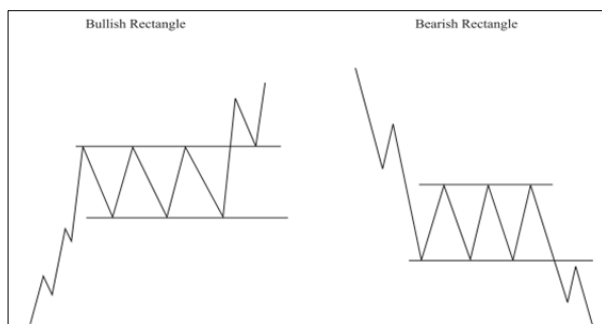
They signal a temporary pause in the trend, and that the trend will continue in the same direction as before. The four kinds of continuation patterns are:

Triangles:

One trendline connects the highs and a second trendline connects the lows. As the distance between the highs and lows narrows, the trendlines converge, forming a triangle. There are three forms - ascending triangles, descending triangles, and symmetric triangles.

Rectangles:

One trendline connects the highs and a second trendline connects the lows. As the distance between the highs and lows is constant, the trendlines are parallel to each other and form a rectangle.



Flags:

They are similar to a rectangle and are formed by two parallel trendlines. However, they form over a much shorter time interval.

Pennants:

They are similar to a triangle and are formed by two converging trend lines. However, they form over a much shorter time interval.

3.4. Technical Indicators

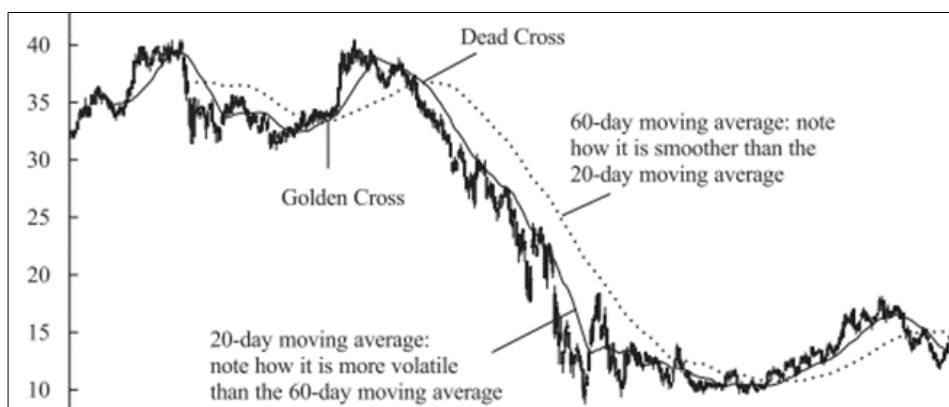
There are four kinds of technical indicators that we will discuss; price-based indicators, momentum oscillators, sentiment indicators and flow-of-funds indicators.

Price-based indicators

They incorporate the information contained in the current and past market prices. The common types are:

Moving average:

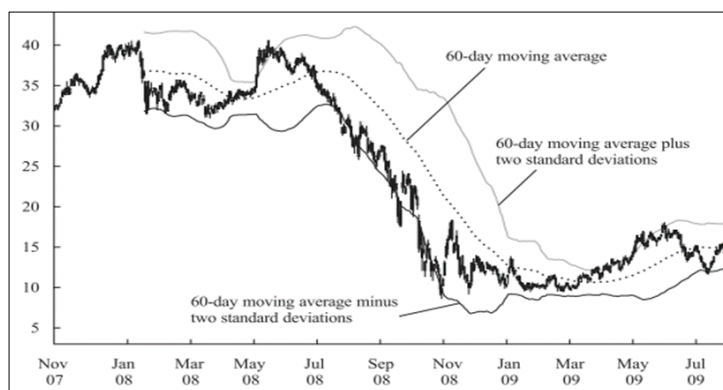
It is the average of the closing prices over a specified number of periods. They are used to smooth out short-term price fluctuations and help identify the trend. When a short-term moving average crosses from underneath a longer-term average, this movement is considered bullish and is known as a golden cross. When a short-term moving average crosses from above a longer-term average, this movement is considered bearish and is known as a dead cross.



Bollinger bands:

Bollinger bands consist of a moving average plus a higher line representing a set number of standard deviations and a lower line representing a set number of standard deviations. The figure below shows a Bollinger band and a moving average.

The more volatile the security becomes, the wider the range becomes between the two outer lines or bands. A common use of a Bollinger band is to create trading strategies such as a contrarian strategy. In this strategy, an investor sells when a security's price reaches the upper band and buys when it reaches the lower band. The contrarian strategy assumes that the security's price will stay within the bands.

**Momentum oscillators**

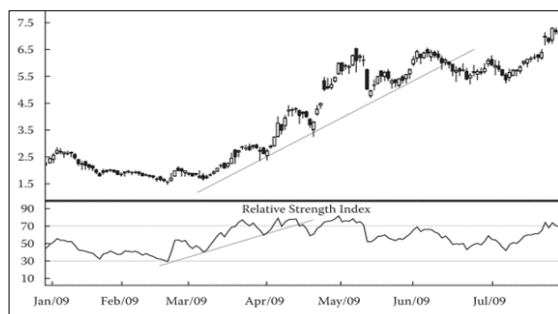
They help to identify changes in the market sentiment. The common types are:

Rate of change (ROC) oscillator:

It oscillates around 0 (or around 100 if an alternative formula is used for calculation). When the ROC oscillator crosses zero into the positive territory, it is considered bullish. When the ROC oscillator crosses zero into the negative territory, it is considered bearish.

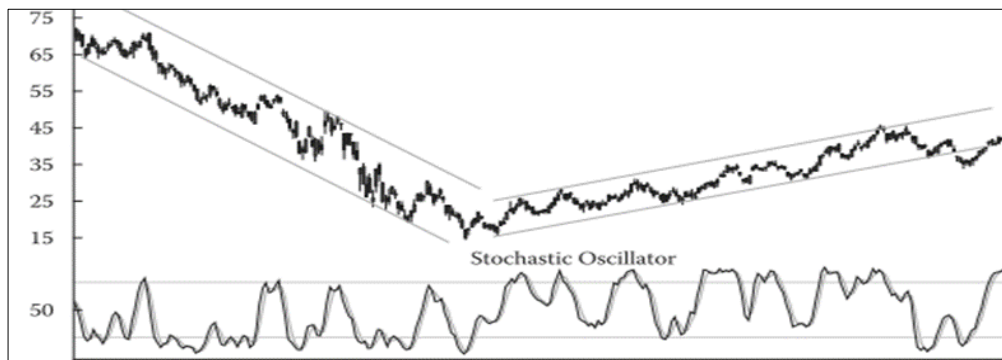
Relative strength index (RSI):

RSI graphically compares a security's gains with its losses over a given period. The popular time period is 14 days. The value of the RSI is always between 0 and 100. A value above 70 represents an overbought situation while a value below 30 suggests that an asset is oversold. An example of an RSI oscillator is given below:

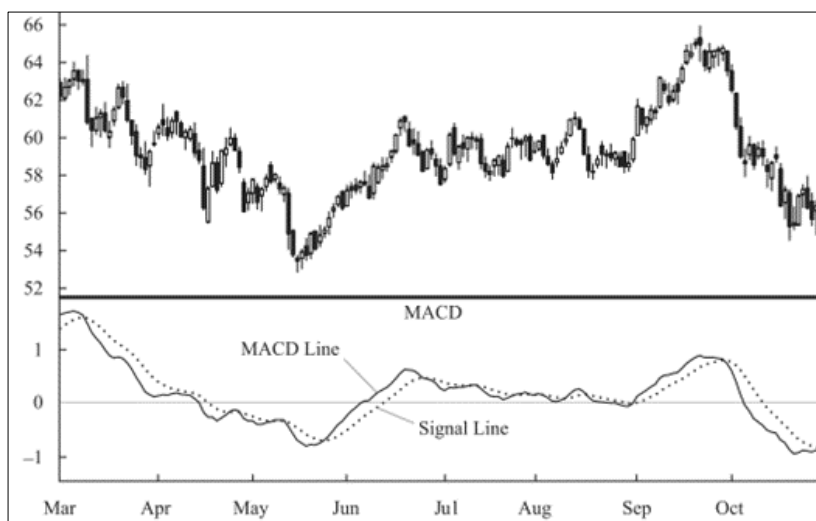


Stochastic oscillator:

It is based on the observation that in uptrends, prices tend to close at or near the high end of their recent range. Similarly, in downtrends, they tend to close near the low end. It is composed of two lines, called %K and %D. It has a default setting of 14-days. It oscillates between 0 and 100. A value above 80 indicates an overbought situation and value below 20 indicates an oversold situation.

Moving-average convergence/divergence oscillator:

It is the difference between a short-term and a long-term moving average of the security's price. It is composed of two lines - MACD line and signal line. It oscillates around 0 and has no upper or lower limit.

**Sentiment indicators**

They gauge investor activity for signs of bullishness or bearishness. The common types are:

Opinion polls:

Regular polls are conducted of investors and investment professionals to gauge the overall market sentiment.

Calculated statistical indices:

- The **put/call ratio** is the volume of put options traded divided by the volume of call options traded. A high ratio indicates that the market is bearish. Whereas, a low ratio indicates that the market is bullish.
- The **CBOE volatility index (VIX)** is a measure of near-term market volatility calculated from option prices of S&P 500 stocks. The VIX rises when market participants become fearful of a market decline.
- **Margin debt** is loans taken by individual investors to fund their stock purchases. When stock margin debt is increasing, investors are aggressively buying and the stock prices will rise because of increased demand.
- **Short interest** refers to the number of shares of a particular security that are currently sold short. The short interest ratio is calculated as:

$$\text{Short interest ratio} = \frac{\text{Short interest}}{\text{Average daily trading volume}}$$

A high ratio suggests an overall negative outlook on the security.

Flow-of-funds indicators

They indicate the change in potential demand and supply. The common types are:

The Arms index: (also known as TRIN)

It is calculated as:

$$\text{Arms index} = \frac{\text{number of advancing issues} \div \text{number of declining issues}}{\text{volume of advancing issues} \div \text{volume of declining issues}}$$

When this index is near 1, the market is in balance. A value above 1 means that there is more volume in declining stocks and that the market is in a selling mood. A value below 1 means that there is more volume in increasing stocks and that the market is in a buying mood.

Margin debt:

Margin loans may increase the purchases of stocks and declining margin balances may force the selling of stocks.

Mutual funds cash position:

Mutual funds must hold some of their assets in cash to pay for miscellaneous expenses and to fund redemptions. During a bullish market, the cash positions tend to be low. During a bearish market, the cash positions tend to be high.

New equity issuance:

IPOs are often timed with bullish markets to get the best valuations. A large number of IPOs may indicate that a market is near its peak.

Secondary offerings:

Like IPOs, technicians also monitor secondary offerings to gauge potential changes in the supply of equities.

3.5. Cycles

Technicians use various cycles to predict future movements in security prices; even cycles in fields such as astronomy and climate can influence the economy and hence capital markets. Commonly referenced cycles are discussed below:

Kondratieff wave: This is the longest of the widely recognized cycles, discovered by Nikolai Kondratieff in the 1920s. He suggested that Western economies have a 54-year old cycle. This cycle is also known as the K Wave.

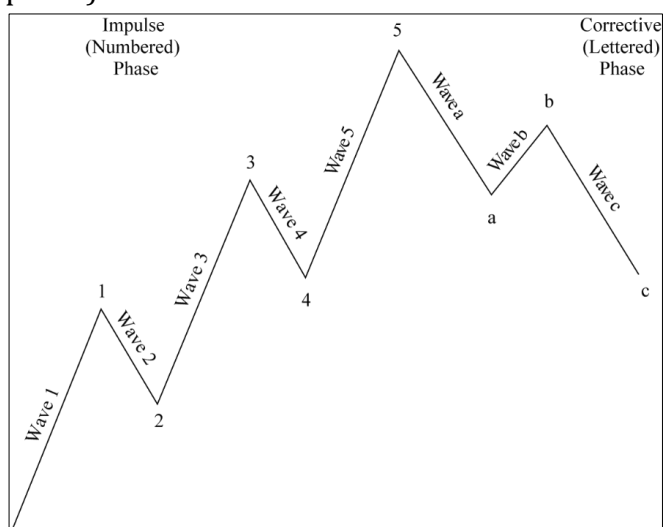
18-year cycle: Three 18-year cycles make up the longer 54-year Kondratieff Wave. The 18-year cycle is often mentioned in connection with real estate prices, but it can also be found in equities and other markets.

Decennial pattern: This pattern connects average stock market returns with the last digit of the year. Years ending in 0 have shown poor performance while years ending in 5 have shown good performance.

Presidential cycle: This cycle in the United States connects the performance of the market with presidential elections. In this theory, the third year following an election shows the best performance.

4. Elliot Wave Theory

According to this theory, the market moves in regular waves or cycles. In a bull market, the market moves up in five waves 1 = up, 2 = down, 3 = up, 4 = down and 5 = up. (Impulse phase) and downward move occurs in three waves 1 = down, 2 = up, 3 = down (Corrective phase).



Each wave can be broken into smaller waves over a shorter time period. Market waves follow patterns that are ratios of the numbers in the Fibonacci sequence. Hence, ratios of the numbers in the Fibonacci sequence can be used to set price targets while trading.

5. Inter-market Analysis

Inter-market analysis is based on the principle that all markets are interrelated and influence each other. Here, technicians look for inflection points in one market as a warning sign to start looking for a change in another related market. To identify these inter-market relationships, a commonly used tool is the relative strength analysis. The relative strength analysis can also be used to identify the strongest performing securities in a sector and to identify the sectors of the equity market to invest in. Lastly, observations based on inter-market analysis can also help in allocating funds across securities from different countries.

Summary

LO.a: Explain principles of technical analysis, its applications, and its underlying assumptions.

Technical analysis is a form of security analysis that involves examination of past price and volume data to predict future behavior of the market or individual security.

Assumptions:

- Market prices are determined by supply and demand.
- Market prices reflect both rational and irrational investor behavior.
- Investor behavior is reflected in trends and patterns that tend to repeat.
- Price and volume information can be used to understand investor sentiment and make investment decisions.

Technical analysis can also be used on assets such as commodities, currencies and futures that do not have underlying income streams or financial statements.

LO.b: Describe the construction of different types of technical analysis charts and interpret them.

Line charts

- Graphic display of prices over time.
- It has only one data point per time interval – the closing price.
- Price is plotted on the Y-axis and time on the X-axis.
- The closing prices for each trading period are connected by a line.

Bar charts

- It has four data points per time interval – opening price, highest and lowest price, and closing price.
- Price is plotted on the Y-axis and time on the X-axis
- They give a better sense of the trend in the market.
- A short bar indicates low volatility, a long bar indicates high volatility

Candlestick charts

- It has the same four data points per time interval as a bar chart– opening price, highest and lowest prices, and closing price.
- Price is plotted on the Y-axis and time on the X-axis.
- If the market closed up, the body of the candle is clear.
- If the market closed down, the body of the candle is shaded.

Volume charts

- Often displayed below a line, bar or candlestick chart.
- Number of units of the security traded is plotted on the Y-axis and time on the X-axis.

Point and figure charts

- Drawn as a grid consisting of columns of X's alternating with columns of O's. X represents an increase in price while an O represents a decrease in price.
- Y-axis measures box size increments in price, whereas X-axis measures a number of price changes.
- To construct this chart, you need to specify a box size and a reversal size.

LO.c: Explain uses of trend, support, resistance lines, and change in polarity.

Uptrend: A security is said to be in an uptrend if prices are reaching higher highs and higher lows. An upward trendline can be drawn by connecting the increasing low points with a straight line.

Downtrend: A security is said to be in a downtrend if prices are reaching lower highs and lower lows. A downward trendline can be drawn by connecting the decreasing high points with a straight line.

Support: It is the price level at which there is sufficient buying pressure to stop a further decline in prices.

Resistance: It is the price level at which there is sufficient selling pressure to stop the further increase in prices.

Change in polarity: Once a support level is breached, it often becomes a new resistance level.

Similarly, once a resistance level is breached; it often becomes a new support level.

LO.d: Describe common chart patterns.

Reversal patterns: They signal the end of a trend. The four kinds of reversal patterns are:

- Head and shoulders pattern
- Inverse head and shoulders pattern
- Double tops and bottoms
- Triple tops and bottoms

Continuation patterns: They signal a temporary pause in the trend, and that the trend will continue in the same direction as before. The four kinds of continuation patterns are:

- Triangles
- Rectangles
- Flags
- Pennants

LO.e: Describe common technical analysis indicators (price-based, momentum oscillators, sentiment, and flow of funds).

Price-based indicators: They incorporate the information contained in the current and past market prices. The common types are:

- Moving average

- Bollinger bands

Momentum oscillators: They help to identify changes in the market sentiment. The common types are:

- Rate of change (ROC) oscillator
- Relative strength index (RSI)
- Stochastic oscillator
- Moving-average convergence/divergence oscillator

Sentiment indicators: They gauge investor activity for signs of bullishness or bearishness. The common types are:

- Opinion polls
- Calculated statistical indices
 - put/call ratio
 - Volatility index (VIX)
 - Margin debt
 - Short interest

Flow-of-funds indicators: They indicate the change in potential demand and supply. The common types are:

- The Arms
- Margin debt
- Mutual funds cash position
- New equity issuance
- Secondary offerings

LO.f: Explain how technical analysts use cycles.

Kondratieff wave (K-Wave)

- It states that Western economies have a 54-year old cycle.

18-year cycle

- Three 18-year cycles make up the longer 54-year Kondratieff wave.
- This cycle is often mentioned in real estate markets, but it can also be found in equities and other markets.

Decennial pattern

- This pattern links average stock market returns with the last digit of the year.
- Years ending in 0 have shown poor performance, whereas years ending in 5 have shown good performance.

Presidential cycle

- This cycle connects the performance of the US market with the US presidential elections.

- Historically, the third year following an election has shown the best performance.

LO.g: Describe the key tenets of Elliott Wave Theory and the importance of Fibonacci numbers.

According to this theory, the market moves in regular waves or cycles. In a bull market, the market moves up in five waves in the following pattern: 1 = up, 2 = down, 3 = up, 4 = down and 5 = up. This wave is known as the impulse wave. Each wave can be broken into smaller waves over a shorter time period. Market waves follow patterns that are ratios of the numbers in the Fibonacci sequence. Hence, the ratios of the numbers in the Fibonacci sequence can be used to set price targets while trading.

LO.h: Describe intermarket analysis as it relates to technical analysis and asset allocation.

Inter-market analysis is based on the principle that different markets such as stocks, bonds, commodities, currencies etc. are interrelated and influence each other. Technicians often use relative strength analysis to look for the inflection point in one market as a warning sign to start looking for a change in another related market. The relative strength analysis can also be used to identify attractive asset classes and attractive sectors within these classes to invest in.

Practice Questions

1. Technical analysis *most likely* relies upon:
 - A. price and volume information.
 - B. fundamental analysis to confirm conclusions.
 - C. financial statements.
 2. Which of the following charts does not show the high and low prices for each trading period?
 - A. Bar chart.
 - B. Point and figure chart.
 - C. Candlestick chart.
 3. An upward trendline is constructed by drawing a line connecting the:
 - A. the highs of the price chart.
 - B. the lows of the price chart.
 - C. the highest high to the lowest low of the price chart.
 4. Which of the following statements is *most likely* explains the principle of 'change in polarity'?
 - A. Once a downtrend is broken it becomes an uptrend.
 - B. The short-term moving average has crossed the long-term moving average.
 - C. Once a support level is breached, it becomes a resistance level.
 5. Lucy, a technical analyst, observes a head and shoulders pattern in a stock she has been following. She notes the following information:

Head Price	\$46.50
Shoulder Price	\$41.50
Neckline Price	\$38.25

Her estimated price target is *closest* to:
 - A. \$21.00.
 - B. \$25.50.
 - C. \$30.00.
 6. Which of the following would a technical analyst *most likely* interpret as a sell signal?
 - A. 100-day moving average crosses below a 50-day moving average.
 - B. 50-day moving average crosses below a 200-day moving average.
 - C. 30-day moving average crosses above a 100-day moving average.
 7. Momentum Oscillators are *most likely* used to:
 - A. indicate an overbought or oversold position.
-

- B. set the target price.
 - C. analyze the movement of price of security with respect to economic changes.
8. A value of 1.2 in the short-term trading index (TRIN) most likely indicates that:
- A. trading volume is heavier in declining issues than in advancing issues.
 - B. trading volume is heavier in advancing issues than in declining issues.
 - C. market is oversold.
9. Kondratieff concluded that western economies generally followed a cycle of how many years?
- A. 44 years.
 - B. 54 years.
 - C. 64 years.
10. In Elliot Wave theory, Fibonacci numbers are used to forecast the:
- A. size of the waves.
 - B. number of subwaves within a larger wave.
 - C. timing of the wave direction change.
11. Which of the following is a *most* commonly used tool for intermarket analysis?
- A. Relative strength analysis.
 - B. Stochastic oscillators.
 - C. Momentum oscillators.

Solutions

1. A is correct. Technical analysis is the study of market trends or patterns and relies upon price and volume data.
2. B is correct. A point and figure chart is basically used to map the change in direction of share prices. Only the opening and closing price is incorporated into point and figure chart. A box is filled with either "x" sign or "o" sign depends on direction of price change. This chart does not incorporate the high and low prices.
3. B is correct. An upward trendline is constructed by drawing a line connecting the lows of the price chart.
4. C is correct.
5. C is correct. Price target = Neckline – (Head – Neckline) = 38.25 – (46.50 – 38.25) = 30
6. B is correct. When using moving averages to generate trading signals, a "golden cross" of a shorter-term average above a longer-term average is a buy signal, while a "dead cross" under the longer-term average is a sell signal.
7. A is correct. The most known use of Momentum Oscillators is to indicate the overbought or oversold position of a security. Thus, it helps in providing signal for buying or selling security, but it does not help to set the target price. Economic conditions, neither affect technical analysis nor are they used in technical analysis.
8. A is correct. The TRIN or Arms index is a flow of funds indicator. Values less than one indicate more trading volume in advancing stocks than in declining stocks, while values greater than one mean more volume is in declining stocks than in advancing stocks.

$$TRIN = \frac{\text{Number of advancing issues} / \text{Number of declining issues}}{\text{Volume of advancing issues} / \text{Volume of declining issues}}$$
9. B is correct. Kondratieff wave is a cycle of 54 years. This is the longest and a widely recognized cycle.
10. A is correct. In Elliot Wave theory, the size of the waves is believed to correspond to the ratio of Fibonacci numbers.
11. A is correct. Relative strength analysis is often used to compare two asset classes or two securities.