Level I Sample Page

Key Facts & Formulas for the CFA®

Quantitative Methods

Three interpretations of interest rates: (1) required rate of return (2) discount rate (3) opportunity cost

Components of Interest Rates
r = real risk-free interest rate + inflation premium + default risk premium + liquidity premium + maturity premium
Nominal risk-free rate = real risk-free rate + inflation premium

Holding Period Return (R): total return for holding an investment over a given time period
$R = \frac{(P_1 - P_0) + I_1}{P_0}$
Holding period return can be computed for a period longer than a year. E.g., holding period return over a 3-year period using three annual returns: R = [(1 + R1) × (1 + R2) × (1 + R3)] - 1

Arithmetic mean: sum of all the observations divided by the total number of observations. $\mu = \frac{\sum_{i=1}^{N} X_i}{N}$

Geometric mean: used to calculate compound growth rate.
$\overline{R}_{Gi} = [(1 + R_1)(1 + R_2) \ldots \ldots (1 + R_T)]^{1/T} - 1$

Harmonic mean: used to find average purchase price for equal periodic investments.
$\overline{X}_H = n / \sum_{i=1}^{n} \left(\frac{1}{X_i}\right)$

Harmonic mean ≤ Geometric mean ≤ Arithmetic mean
Arithmetic mean × Harmonic mean = (Geometric mean)^2

Internal rate of return (IRR): the discount rate at which the sum of present values of cash flows will equal zero.

Money weighted rate of return (MWRR): the IRR of a project. It reflects the actual return earned on an investment after accounting for the value and timing of cash flows relating to the investment.

Time weighted rate of return (TWRR): compound growth rate at which $1 invested in a portfolio grows over a given measurement period. The geometric mean formula is used. The TWRR is not impacted by timing and amount of cashflows and is therefore preferred as a performance measure if the portfolio manager does not control the timing and amount of the investment.

Gross return: the return earned by an asset manager prior to deducting management fees and taxes. It measures investment skill.
Net return: accounts for all managerial and administrative expenses. It is what the investor is concerned with.
Pre-tax nominal return: the return before accounting for inflation and taxes; this is the default, unless otherwise stated.
After-tax nominal return: the return after accounting for taxes.
Real return: the return after accounting for taxes and inflation.

Future value: value to which an investment will grow after one or more compounding periods.
$FV_t = PV(1 + r)^t$

If $t \to \infty$: $FV_t = PVe^{rt}$

Present value: current value of some future cash flow.
$PV = FV_t \left[\frac{1}{(1+r)^t}\right]$
If $t \to \infty$: $PV_t = FVe^{-rt}$
Non-annual compounding: $FV = PV / (1 + R_s/m)^{mN}$ where m = number of compounding periods per year, R_s = quoted annual interest rate, and N = number of years

Annualized Return: $R_{annual} = (1 + R_{period})^c - 1$
where c = number of periods in a year

Continuously compounded return: $r_{0,T} = \ln(P_T/P_0)$

Return on a leveraged portfolio:
$R_L = \frac{Portfolio\ Return}{Portfolio\ Equity} = R_P + \frac{V_B}{V_E}(R_P - r_D)$

Perpetuity: never ending series of equal cash flows at regular intervals: PV = PMT / r

Annuity: series of equal cash flows at regular intervals:

$A = \frac{r(PV)}{1-(1+r)^{-t}}$

Cash flow additivity principle: only cash flows occurring at the same point in time can be added. This principle ensures that market prices reflect the condition of no arbitrage.

Measures of Central Tendency
Sample mean: the arithmetic mean of a sample, where n is the number of observations in the sample:
$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$
Mode: Most frequently occurring value in a distribution.
Median: Midpoint of a data set that has been sorted into ascending or descending order.
Position of a percentile in a data set: $L_y = (n+1)\ y\ /100$
Box and whiskers plot: used to visualize the dispersion of data across quartiles.

Measures of Dispersion
Range = maximum value – minimum value
Mean absolute deviation (MAD): average of the absolute values of deviations from the mean.
$MAD = [\sum_{i=1}^{n} |X_i - \overline{X}|]/n$
Variance: mean of the squared deviations from the arithmetic mean
Population variance $\sigma^2 = \sum_{i=0}^{N} (X_i - \mu)^2 / N$
Standard deviation: square root of variance
Target downside deviation (target semideviation): A measure of the risk of being below a given target.

$$S_{Target} = \sqrt{\sum_{\text{for all } X_i \le B}^{n} \frac{(X_i - B)^2}{n - 1}}$$

Coefficient of variation: measures the risk per unit of return; lower value is better. $CV = \frac{s}{\bar{x}}$

Skewness
Symmetrical distribution: mean = median = mode
Positively skewed (long tail on right side): mean > median > mode
Negatively skewed (long tail on left side): mean < median < mode

Kurtosis
Excess kurtosis = kurtosis - 3.
Leptokurtic distribution: Fatter tails than a normal distribution and an excess kurtosis > 0.
Platykurtic distribution: Thinner tails than a normal distribution and an excess kurtosis < 0.
Mesokurtic distribution: identical to a normal distribution and has an excess kurtosis = 0.